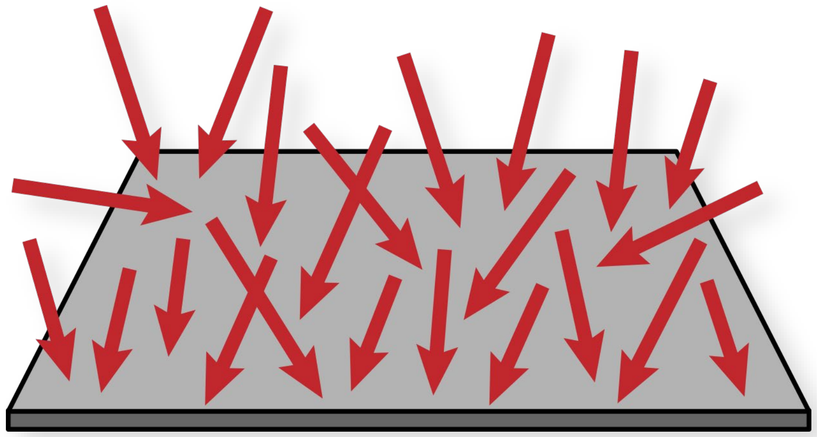
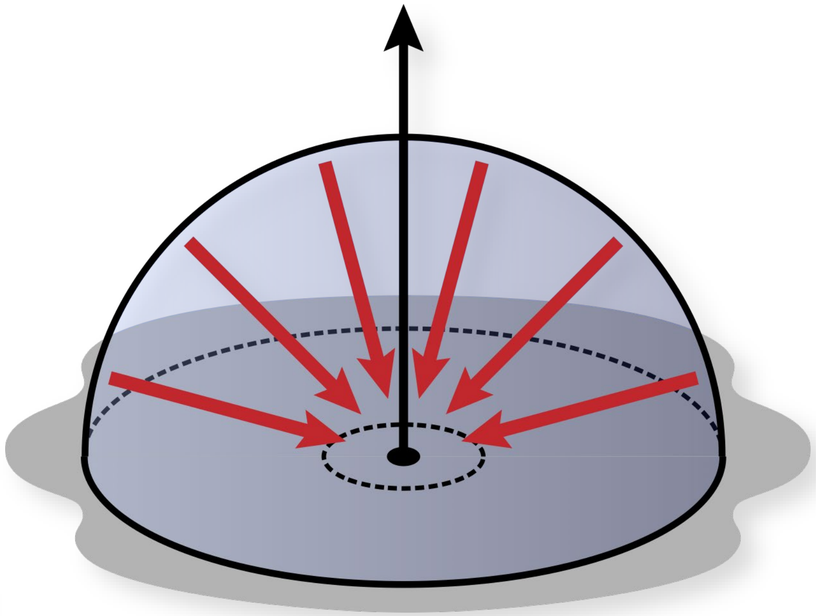
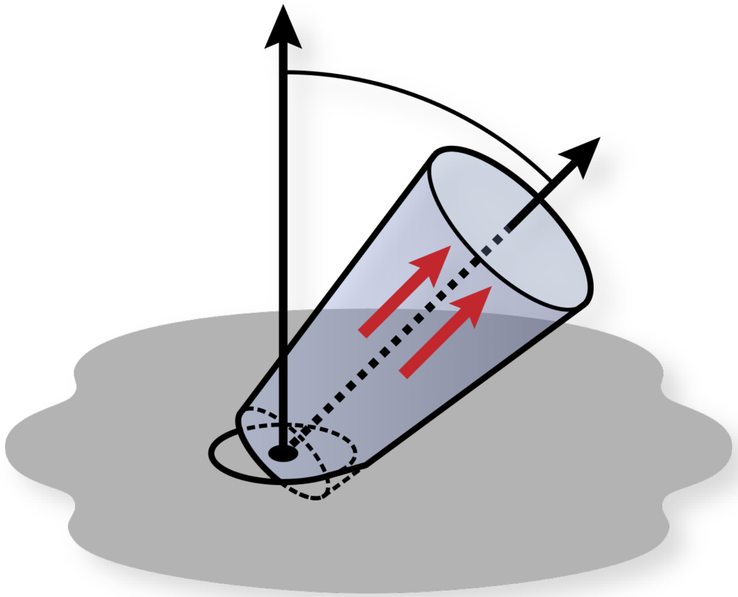
Radiometry and reflectance







15-463, 15-663, 15-862
Computational Photography
Fall 2021, Lecture 14

http://graphics.cs.cmu.edu/courses/15-463

Course announcements

- Homework assignment 4 due November 1st.
  - Generally shorter to accommodate final project proposals.
  - Two bonus parts.
  - Download updated version.

- Homework assignment 5 will be posted tonight.

- Office hour logistics for this week:
  - Yannis will have extra office hours on Friday 3 - 5 pm.

- Mid-semester survey: https://docs.google.com/forms/d/e/1FAIpQLSez9X21VZfQqIBy0tqfHHbrYo5sC6Q4QIYxt3SHva_vpTvy8Q/viewform

Mid-semester survey

- “Homework very hard LOL”.
- Make writeups clearer and remove the need for FAQs.
- Update FAQs faster.
- Clearly mark questions that need answers in homework assignments.
- Ask fewer questions during lectures.
- Slower pace when discussing math, to better absorb content.
- Better visibility for whiteboard.
- Make whiteboard visible in recordings.
- Use Panopto (?).
- Piazza is redundant.
- Pin good threads/discussions on Slack.
- Make Slack more casual.
- Shift office hours to Monday.
- Better align homework assignments and lectures.
- Math refresher for things like signal processing, probability, optimization.

Overview of today's lecture

- Radiometric quantitites.
- A little bit about color.
- Reflectance equation.
- Standard reflectance functions.

Slide credits

Most of these slides were directly adapted from:

- Wojciech Jarosz (Dartmouth).
- Todd Zickler (Harvard).
- Srinivasa Narasimhan (CMU).

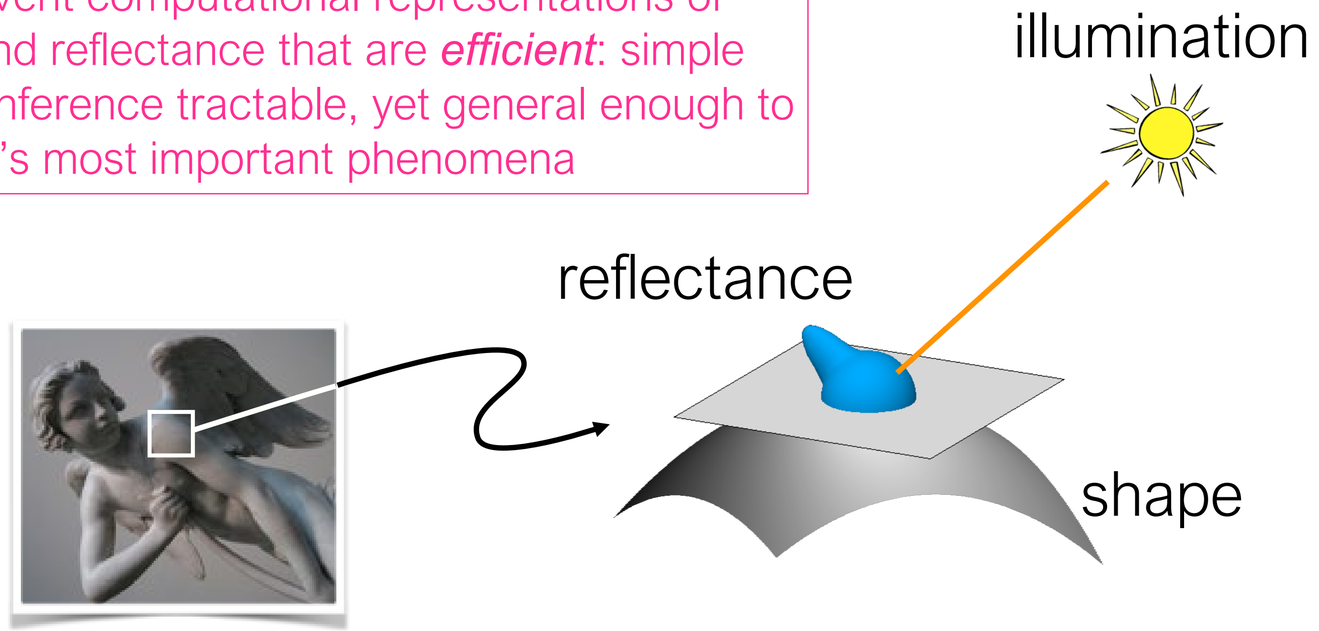
Appearance

Appearance



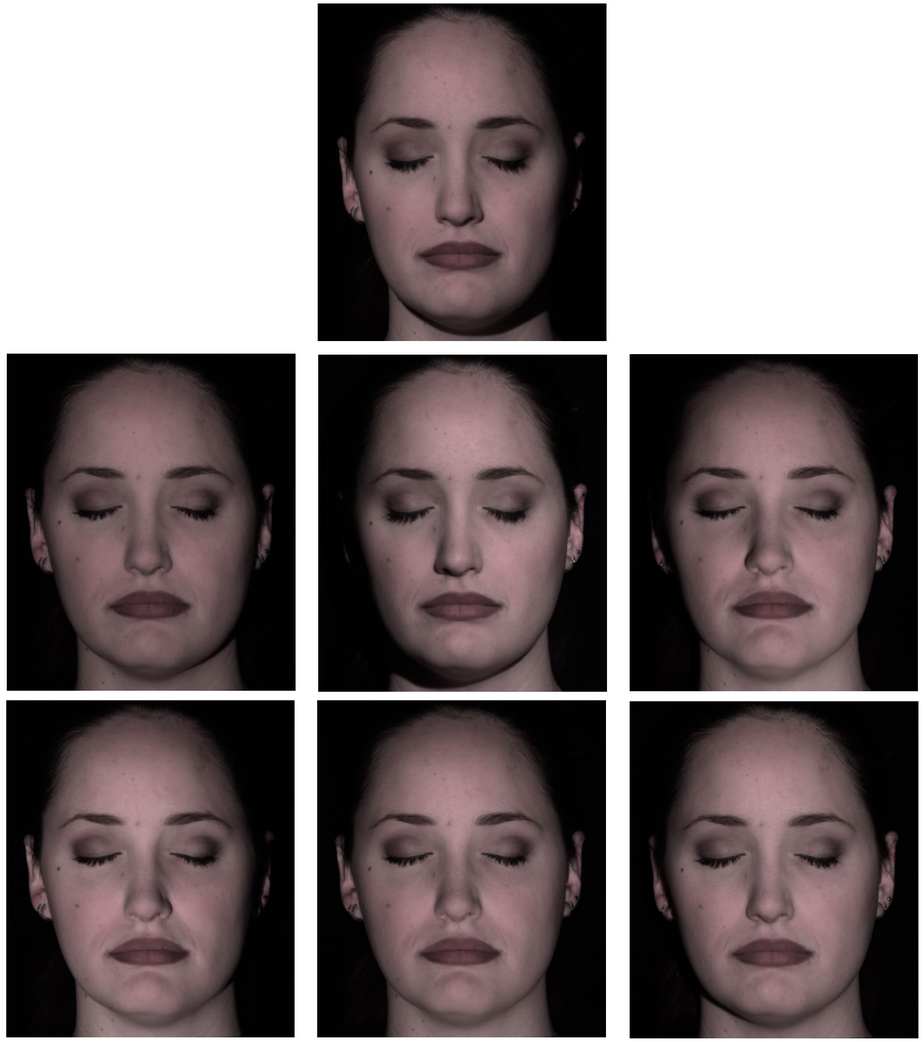
“Physics-based” computer vision (a.k.a “inverse optics”)

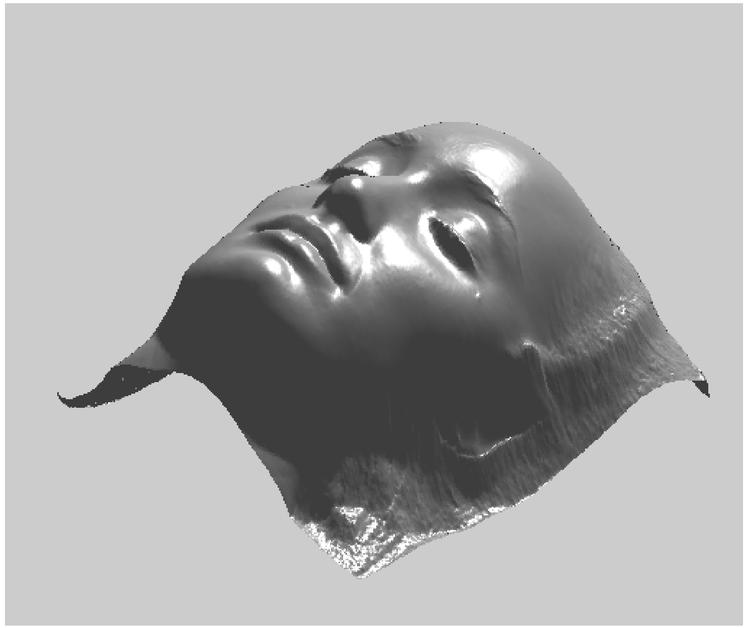
Our challenge: Invent computational representations of shape, lighting, and reflectance that are ***efficient***: simple enough to make inference tractable, yet general enough to capture the world’s most important phenomena



$\mathbf{I} \Longrightarrow$ shape, illumination, reflectance

Example application: Photometric Stereo





Light and Shadows







Reflections







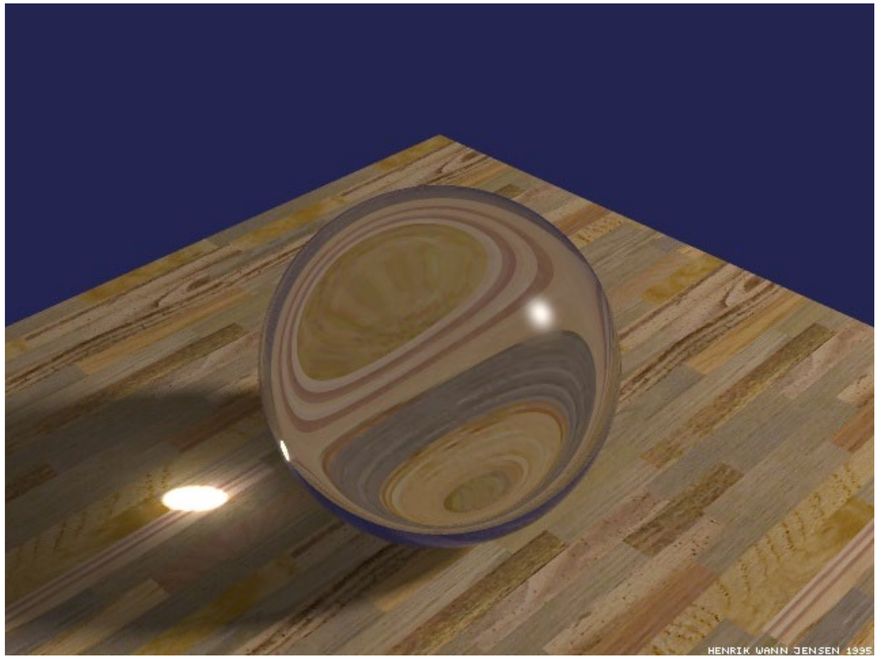




Refractions




HENRIK WANN JENSEN 1995

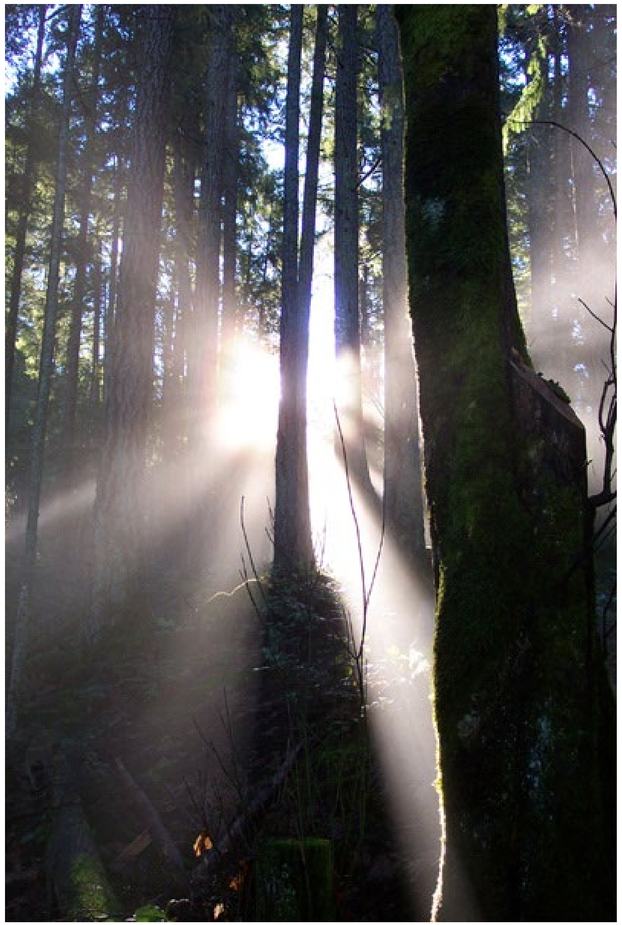

HENRIK WANN JENSEN 1995

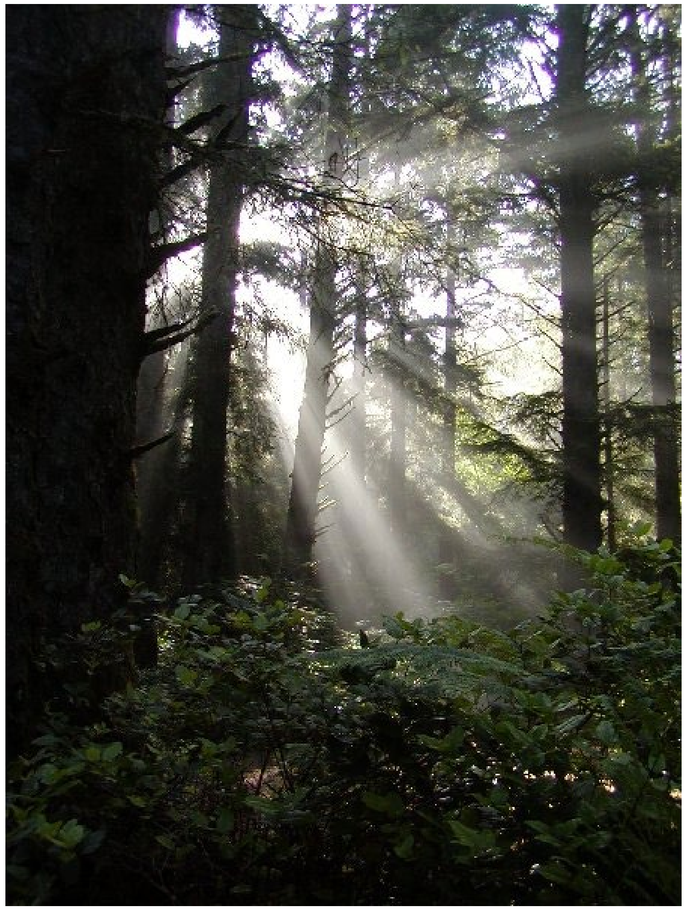


Interreflections


Mies Courtyard House with Curved Elements

Scattering









More Complex Appearances


RENDERED BY HENRIK WANN JENSEN - 2001


RENDERED BY HENRIK WANN JENSEN - 2001





















Quantifying Light

Assumptions

Light sources, reflectance spectra, sensor sensitivity modeled separately at each wavelength

Geometric/ray optics

No polarization

No fluorescence, phosphorescence, …

Radiometry

Radiometry studies the measurement of electromagnetic radiation, including visible light.



Radiometry

Assume light consists of photons with:

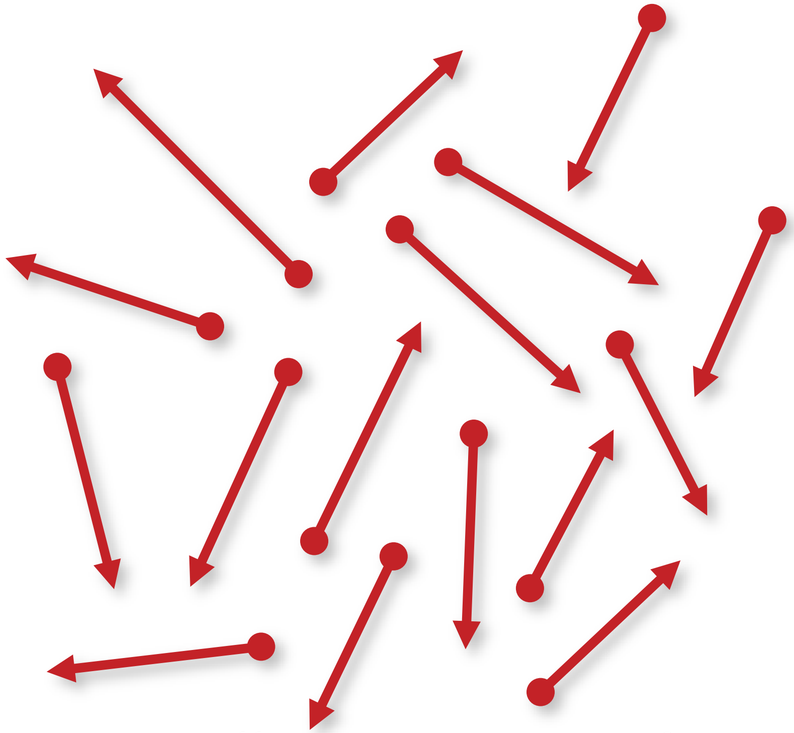
- $\mathbf{x}$: Position
- $\vec{\omega}$: Direction of travel
- λ: Wavelength

Each photon has an energy of: $\frac{h\,c}{\lambda}$

- $h \approx 6.63 \times 10^{-34}\,\mathrm{m^2\,kg/s}$: Planck's constant
- $c = 299{,}792{,}458\,\mathrm{m/s}$: speed of light in vacuum
- Unit of energy, Joule: $\left[\mathrm{J} = \mathrm{kg\,m^2/s^2}\right]$

Radiometry

How do we measure the energy flow?

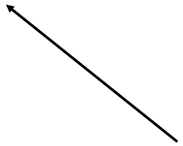


Measuring energy means “counting photons”

Radiometry

Basic quantities (depend on wavelength)

- flux Φ
- irradiance E
- radiosity B
- intensity I
- radiance L

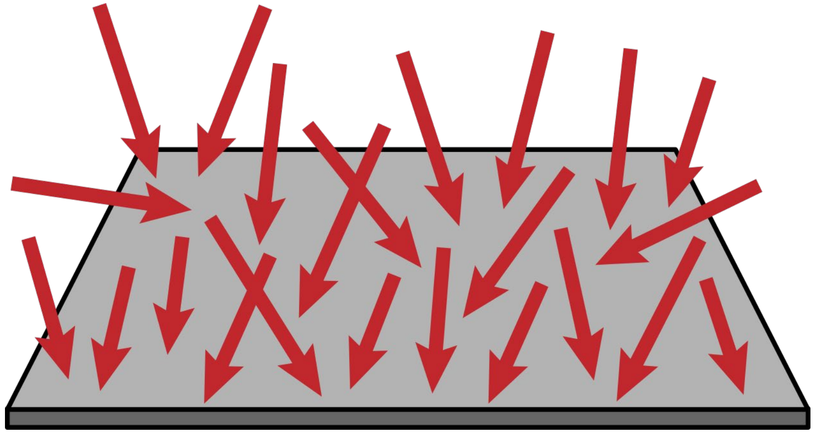


will be the most important quantity for us

Flux (Radiant Flux, Power)

total amount of radiant energy passing through surface or space *per unit time*

$$\Phi(A) \qquad \left[\frac{\mathrm{J}}{\mathrm{s}} = \mathrm{W}\right]$$



examples:

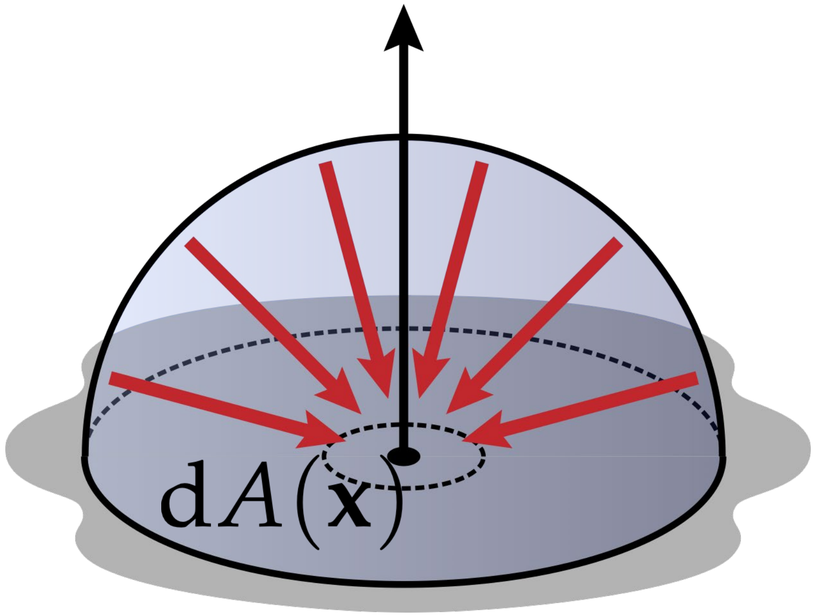
- number of photons hitting a wall per second
- number of photons leaving a lightbulb per second (how do we quantify this exactly?)

Irradiance

area density of flux

flux per unit area **arriving** at a surface

$$E(\mathbf{x}) = \frac{\mathrm{d}\Phi(A)}{\mathrm{d}A(\mathbf{x})} \quad \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]$$



example:

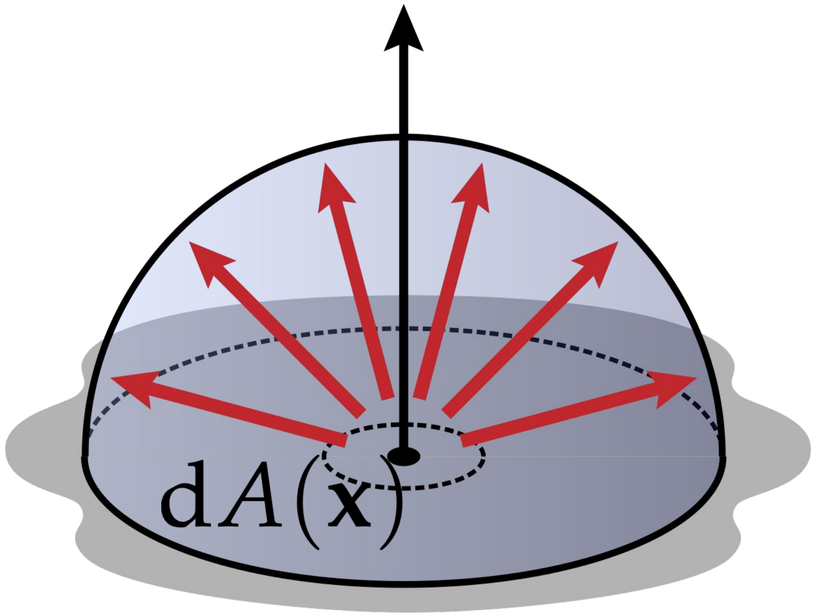
- number of photons **hitting** a small patch of a wall per second, *divided* by size of patch

Radiosity (Radiant Exitance)

area density of flux

flux per unit area **leaving** a surface

$$B(\mathbf{x}) = \frac{\mathrm{d}\Phi(A)}{\mathrm{d}A(\mathbf{x})} \quad \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]$$



example:

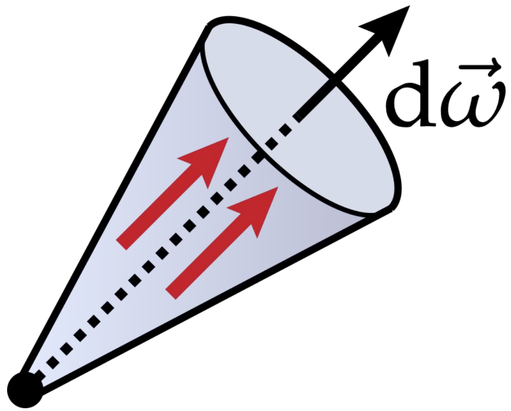
- number of photons **reflecting off** a small patch of a wall per second, *divided* by size of patch

Radiant Intensity

directional density of flux

power (flux) per solid angle

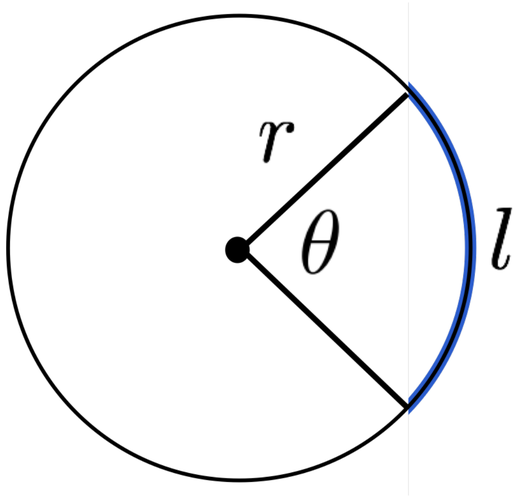
$$I(\vec{\omega}) = \frac{\mathrm{d}\Phi}{\mathrm{d}\vec{\omega}} \qquad \left[\frac{\mathrm{W}}{\mathrm{sr}}\right]$$



Solid Angle

Angle

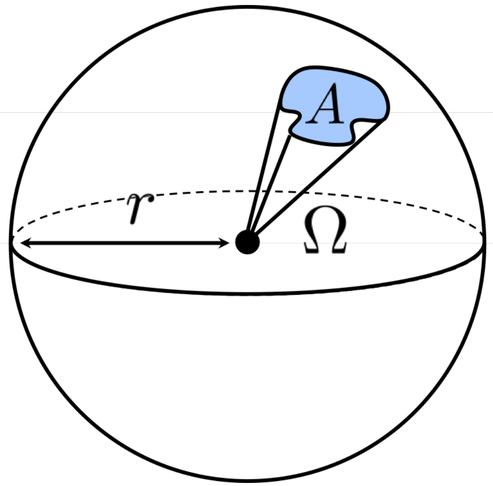
- circle: 2π radians



$$\theta = \frac{l}{r}$$

Solid angle

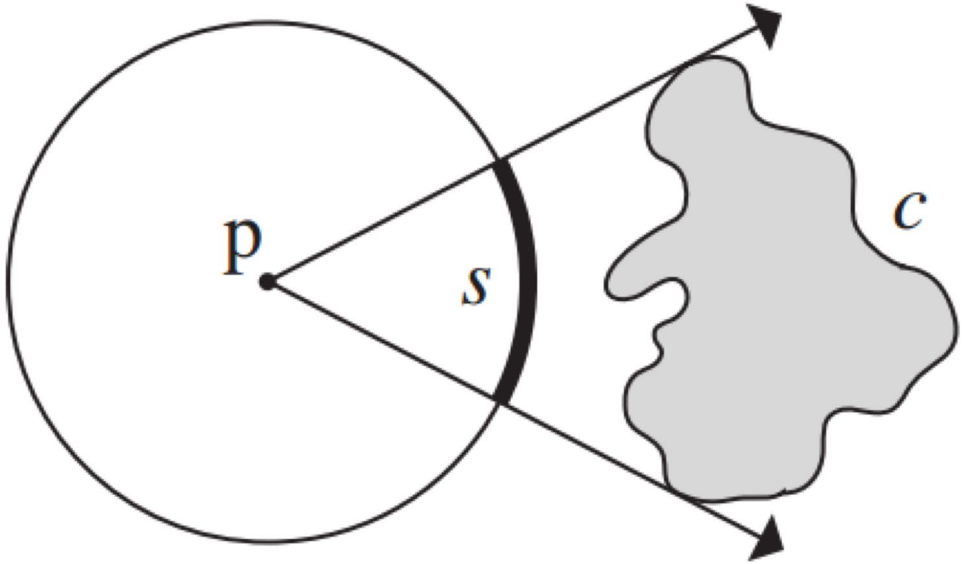
- sphere: 4π steradians

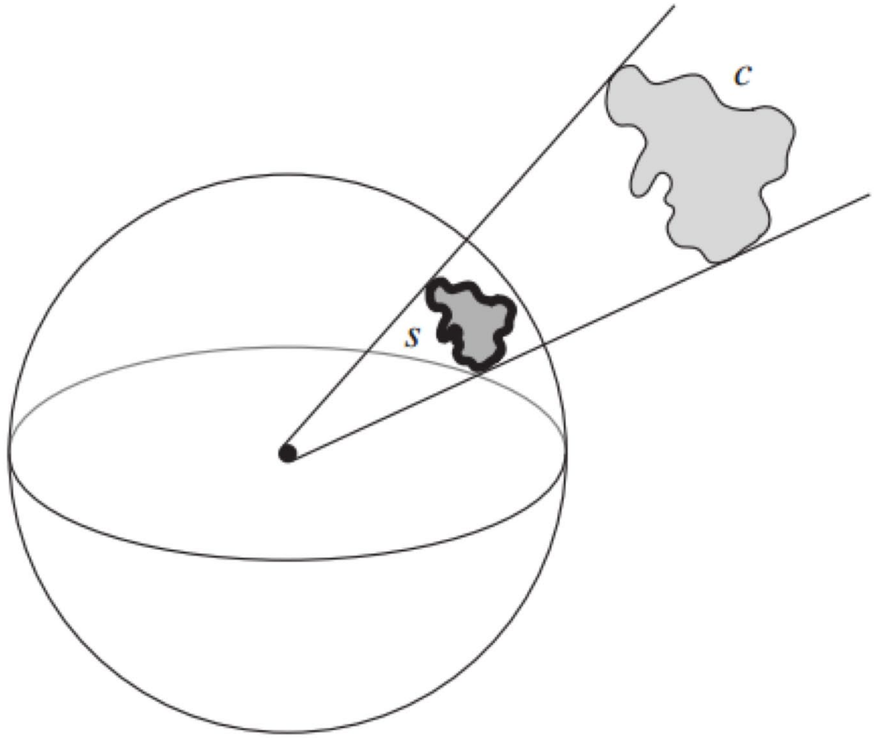


$$\Omega = \frac{A}{r^2}$$

Subtended (Solid) Angle

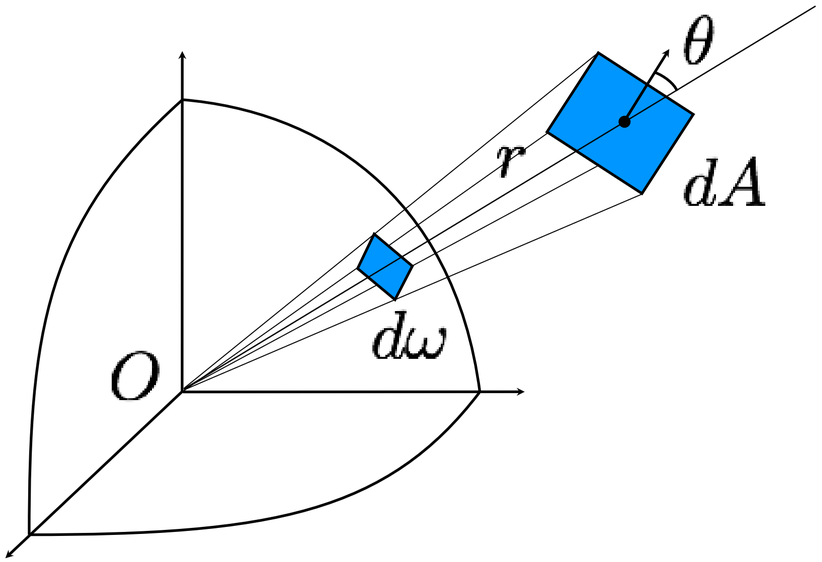
Length/area of object's *projection* onto a unit circle/sphere





Solid angle

The *solid angle* subtended by a small surface patch with respect to point O is the area of its central projection onto the unit sphere about O



Depends on:

- orientation of patch
- distance of patch

One can show:

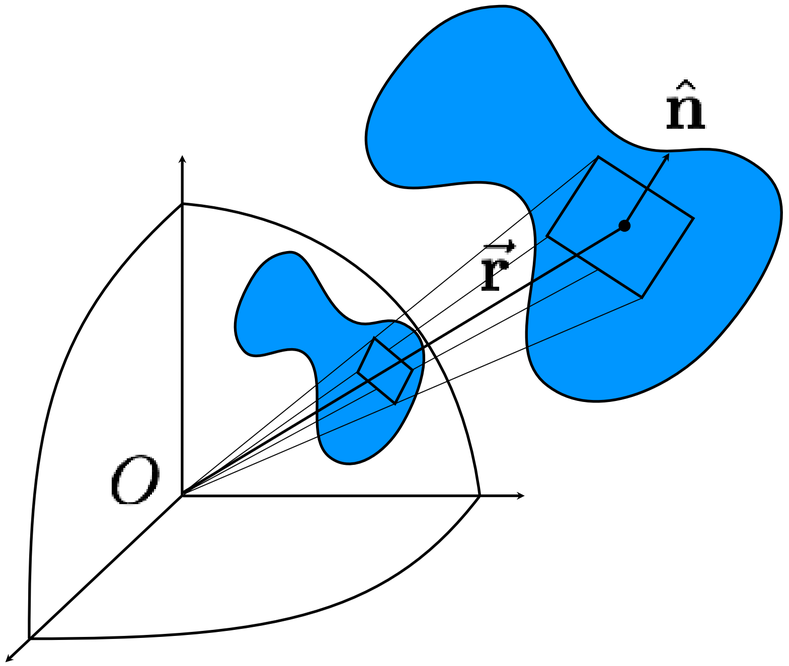
"surface foreshortening"

$$d\omega = \frac{dA\cos\theta}{r^2}$$

Units: steradians [sr]

Solid angle

To calculate solid angle subtended by a surface S relative to O you must add up (integrate) contributions from all tiny patches (nasty integral)



$$\Omega = \iint_S \frac{\vec{\mathbf{r}} \cdot \hat{\mathbf{n}}\, dS}{|\vec{\mathbf{r}}|^3}$$

One can show:

"surface foreshortening"

$$d\omega = \frac{dA \cos\theta}{r^2}$$

Units: steradians [sr]

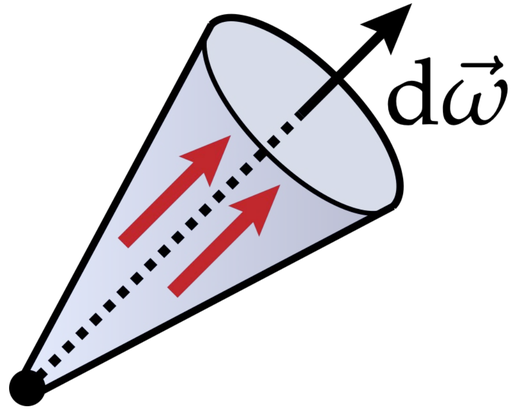
Radiant Intensity

directional density of flux

power (flux) per solid angle

$$I(\vec{\omega}) = \frac{\mathrm{d}\Phi}{\mathrm{d}\vec{\omega}} \qquad \left[\frac{\mathrm{W}}{\mathrm{sr}}\right]$$

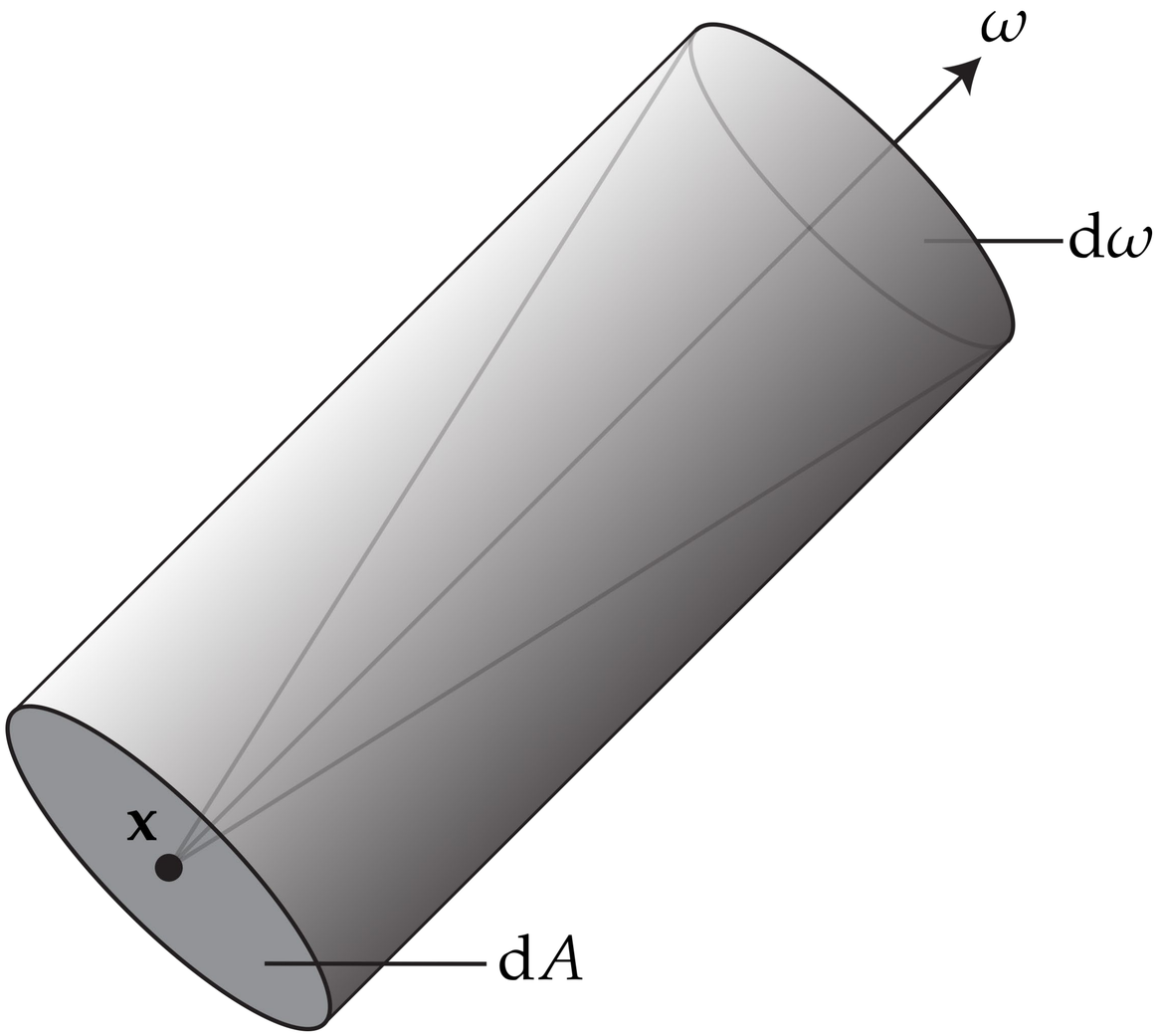
$$\Phi = \int_{S^2} I(\vec{\omega})\,\mathrm{d}\vec{\omega}$$



example: $\Phi = 4\pi I$ (for an isotropic point source)

- power per unit solid angle emanating from a point source

A hypothetical measurement device

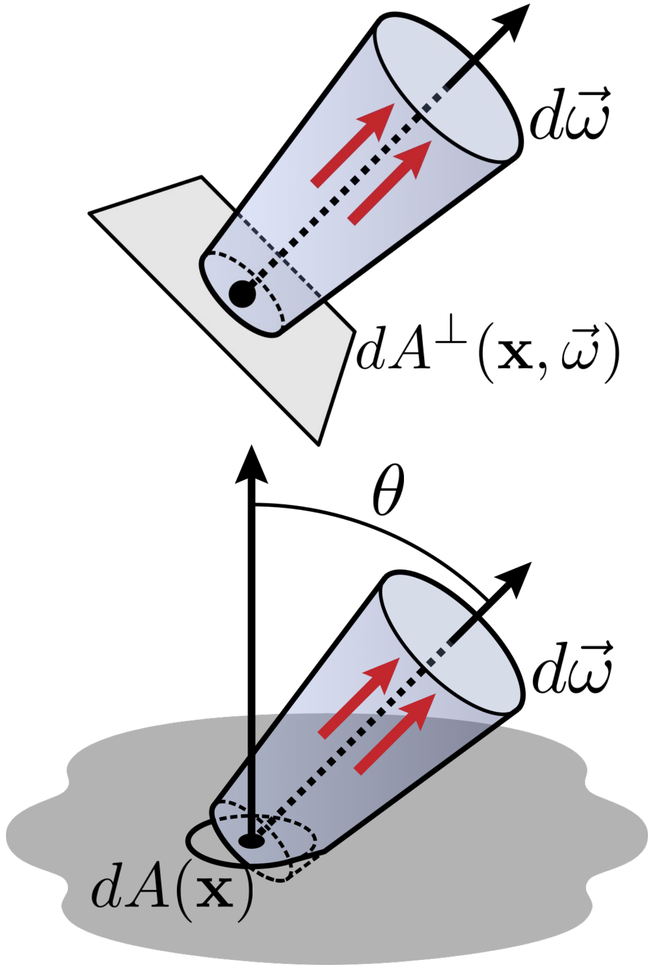


Radiance

flux density per unit solid angle, per *perpendicular* unit area

$$L(\mathbf{x}, \vec{\omega}) = \frac{d^2\Phi(A)}{d\vec{\omega} dA^{\perp}(\mathbf{x}, \vec{\omega})} \left[\frac{W}{m^2 sr}\right]$$

$$= \frac{d^2\Phi(A)}{d\vec{\omega} dA(\mathbf{x}) \cos\theta}$$



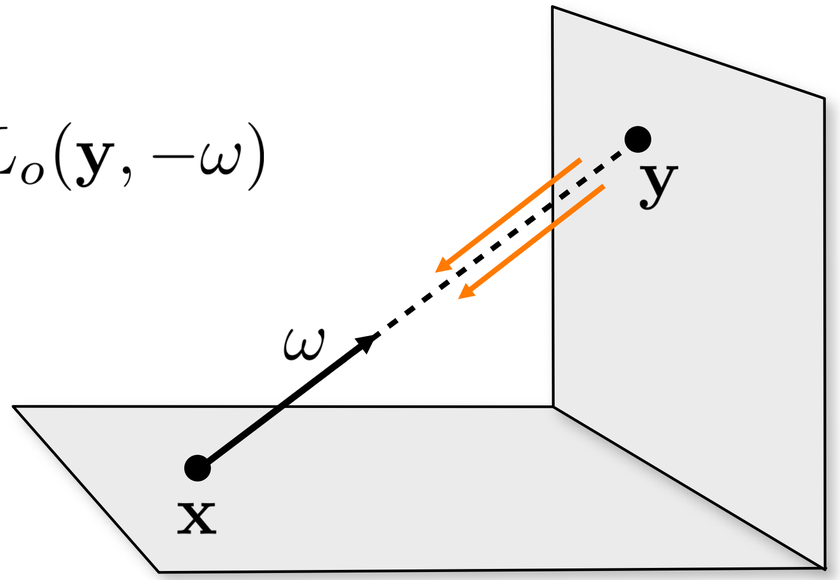
Radiance

fundamental quantity for vision and graphics

remains constant along a ray (*in vacuum only*!)

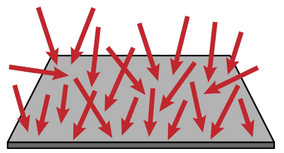
incident radiance L_i at one point can be expressed as outgoing radiance L_o at another point

$$L_i(\mathbf{x}, \omega) = L_o(\mathbf{y}, -\omega)$$

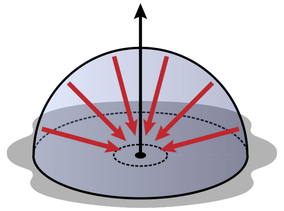


Overview of Quantities

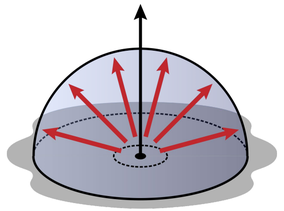
- flux: $\Phi(A)$ $\left[\frac{J}{s} = W\right]$



- irradiance: $E(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}$ $\left[\frac{W}{m^2}\right]$



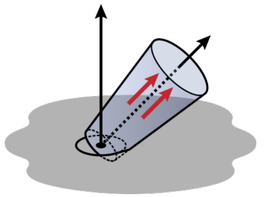
- radiosity: $B(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}$ $\left[\frac{W}{m^2}\right]$



- intensity: $I(\vec{\omega}) = \frac{d\Phi}{d\vec{\omega}}$ $\left[\frac{W}{sr}\right]$



- radiance: $L(\mathbf{x}, \vec{\omega}) = \frac{d^2\Phi(A)}{\cos\theta dA(\mathbf{x}) d\vec{\omega}}$ $\left[\frac{W}{m^2 sr}\right]$



Radiance

expressing *irradiance* in terms of radiance:

$$L(\mathbf{x}, \vec{\omega}) = \frac{d^2\Phi(A)}{\cos\theta dA(\mathbf{x}) d\vec{\omega}} \qquad E(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}$$

$$L(\mathbf{x}, \vec{\omega}) = \frac{dE(\mathbf{x})}{\cos\theta d\vec{\omega}}$$

$$L(\mathbf{x}, \vec{\omega}) \cos\theta \, d\vec{\omega} = dE(\mathbf{x})$$

$$\int_{H^2} L(\mathbf{x}, \vec{\omega}) \cos\theta \, d\vec{\omega} = E(\mathbf{x})$$

Integrate cosine-weighted radiance over hemisphere

Radiance

expressing *irradiance* in terms of radiance:

$$\int_{H^2} L(\mathbf{x}, \vec{\omega}) \cos\theta \, d\vec{\omega} = E(\mathbf{x})$$

expressing *flux* in terms of radiance:

$$\int_A E(\mathbf{x}) \, dA(\mathbf{x}) = \Phi(A) \qquad E(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}$$

$$\int_A \int_{H^2} L(\mathbf{x}, \vec{\omega}) \cos\theta \, d\vec{\omega} dA(\mathbf{x}) = \Phi(A)$$

Integrate cosine-weighted radiance
over hemisphere and area

Radiance

Allows computing the radiant flux measured by *any* sensor

$$\Phi(W, X) = \int_X \int_W L(\hat{\boldsymbol{\omega}}, x) \cos\theta d\boldsymbol{\omega} dA$$

Cameras measure integrals of radiance (after a one-time radiometric calibration). So RAW pixel values are proportional to (integrals of) radiance.

- “Processed” images (like PNG and JPEG) are not linear radiance measurements!!

Computing spherical integrals

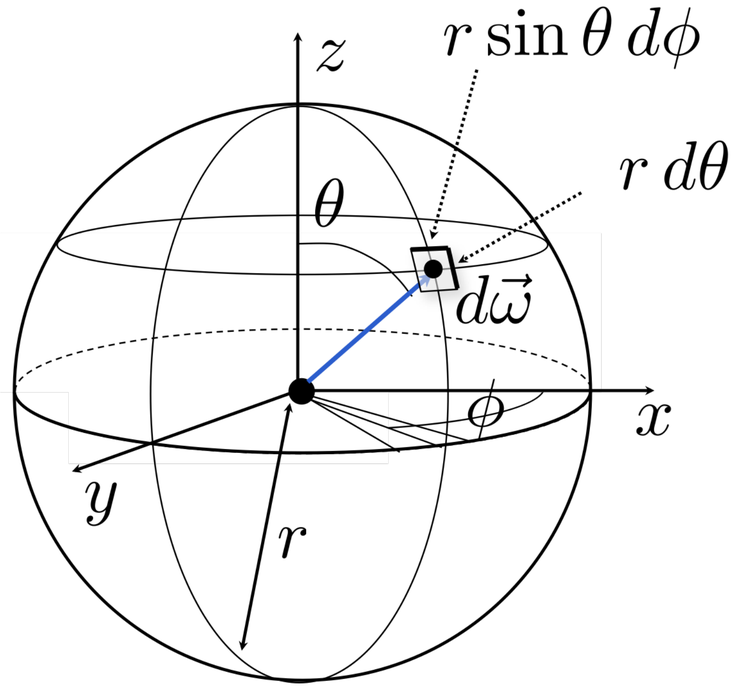
Express function using spherical coordinates:

$$\int_0^{2\pi}\int_0^{\pi} f(\theta,\phi)\ \mathrm{d}\theta\,\mathrm{d}\phi\ \ \textbf{?}$$

Warning: this is not correct!

Differential Solid Angle

Differential area on the unit sphere around direction $\vec{\omega}$



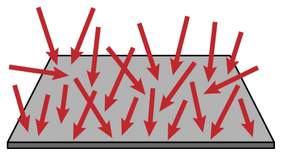
$$dA = (rd\theta)(r\sin\theta d\phi)$$

$$d\vec{\omega} = \frac{dA}{r^2} = \sin\theta d\theta d\phi$$

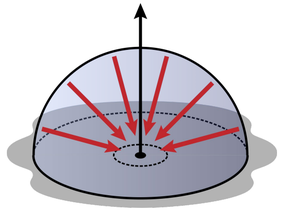
$$\Omega = \int_{S^2} d\vec{\omega} = \int_0^{2\pi}\int_0^{\pi} \sin\theta d\theta d\phi = 4\pi$$

Overview of Quantities

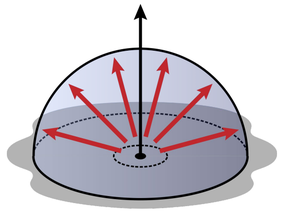
- flux: $\Phi(A)$ $\left[\frac{J}{s} = W\right]$



- irradiance: $E(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}$ $\left[\frac{W}{m^2}\right]$



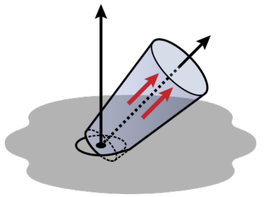
- radiosity: $B(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}$ $\left[\frac{W}{m^2}\right]$



- intensity: $I(\vec{\omega}) = \frac{d\Phi}{d\vec{\omega}}$ $\left[\frac{W}{sr}\right]$



- radiance: $L(\mathbf{x}, \vec{\omega}) = \frac{d^2\Phi(A)}{\cos\theta dA(\mathbf{x}) d\vec{\omega}}$ $\left[\frac{W}{m^2 sr}\right]$



All of these quantities can be a function of wavelength!

Handling color

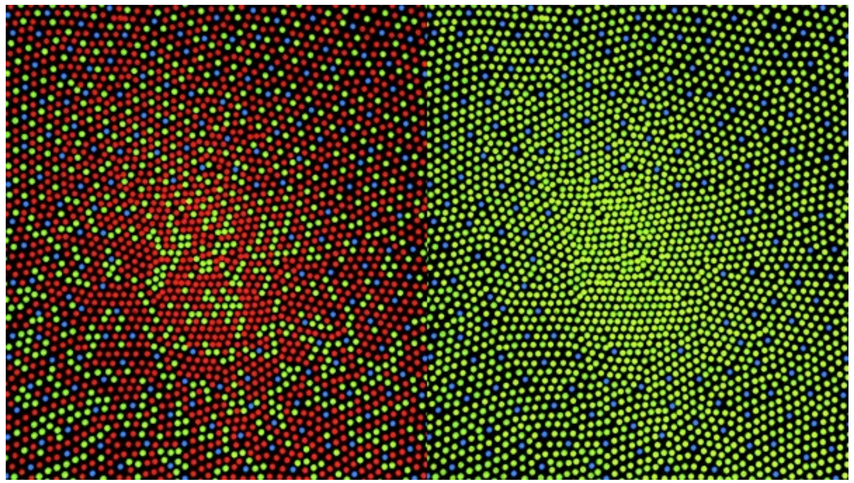
- *Any* light sensor (digital or not) has different sensitivity to different wavelengths.
- This is described by the sensor's *spectral sensitivity function* (SSF).
- When measuring some incident *spectral* flux, the sensor produces a *scalar color* response:

spectral flux sensor SSF

sensor response $\longrightarrow R = \int_{\lambda} \Phi(\lambda) f(\lambda) d\lambda$

Handling color – the human eye

- The human eye is a collection of light sensors called cone cells.
- There are three types of cells with different spectral sensitivity functions.
- Human color perception is three-dimensional (*tristimulus color*).

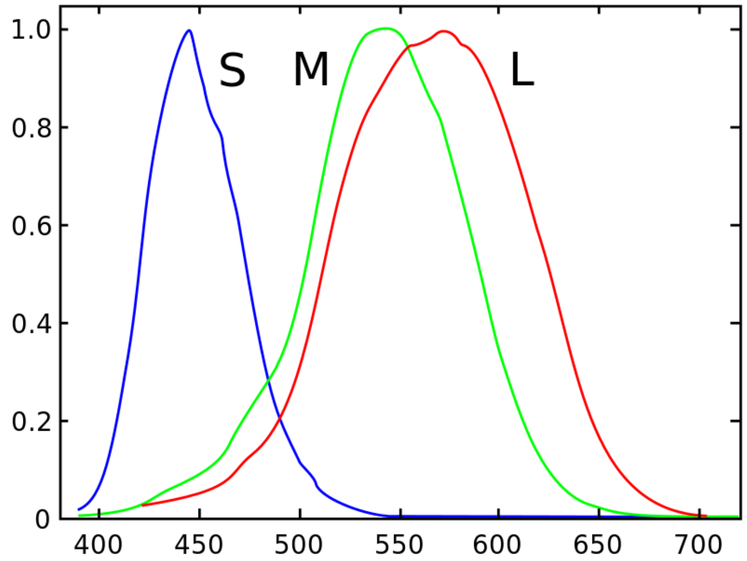


cone distribution
for normal vision
(64% L, 32% M)

"short" $S = \int_\lambda \Phi(\lambda) S(\lambda) d\lambda$

"medium" $M = \int_\lambda \Phi(\lambda) M(\lambda) d\lambda$

"long" $L = \int_\lambda \Phi(\lambda) L(\lambda) d\lambda$

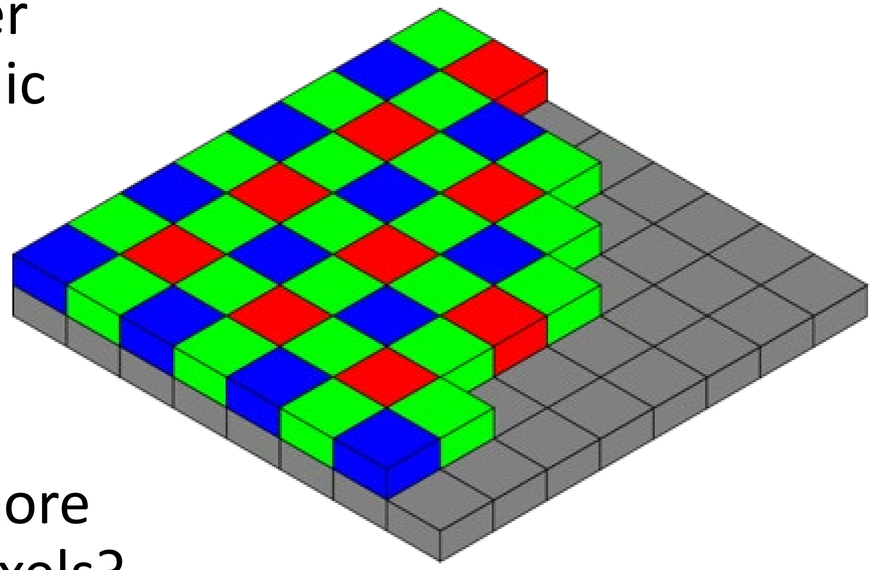


Handling color – photography

Two design choices:

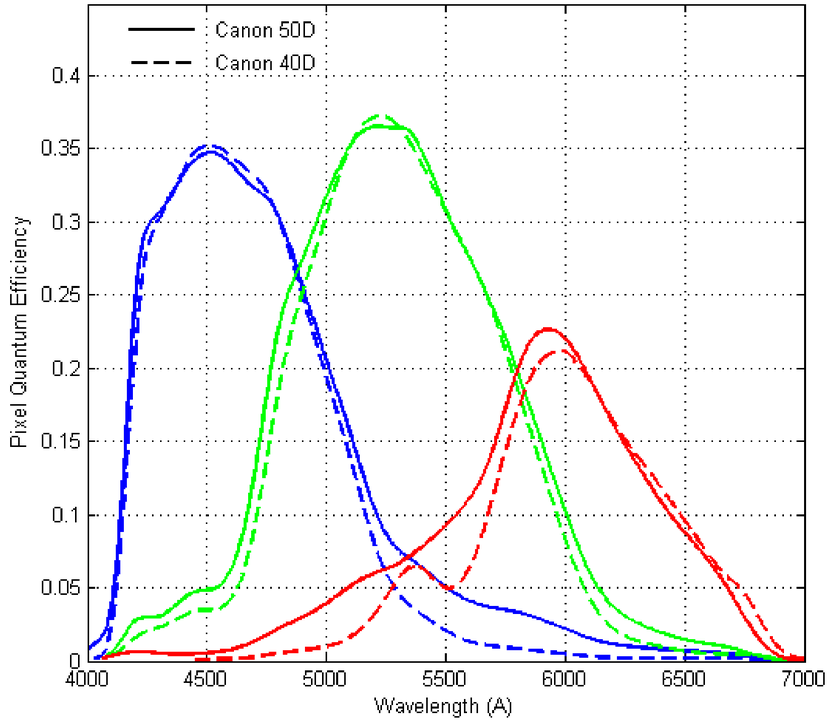
- What spectral sensitivity functions $f(\lambda)$ to use for each color filter?
- How to spatially arrange ("mosaic") different color filters

Bayer mosaic



Why more green pixels?

SSF for Canon 50D

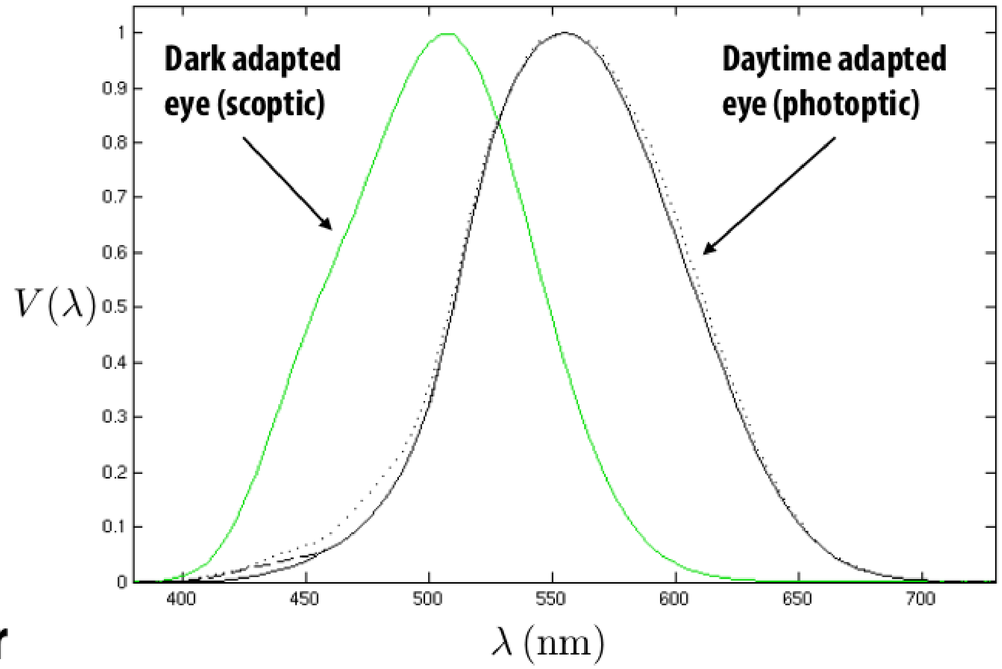


Generally do not match human LMS.

$f(\lambda)$

Radiometry versus photometry

- All radiometric quantities have equivalents in photometry
- Photometry: accounts for response of human visual system to electromagnetic radiation
- Luminance (Y) is photometric quantity that corresponds to radiance: integrate radiance over all wavelengths, weight by eye's luminous efficacy curve, e.g.:



$$Y(\mathrm{p},\omega) = \int_0^\infty L(\mathrm{p},\omega,\lambda)\, V(\lambda)\, \mathrm{d}\lambda$$

Radiometry versus photometry

Physics	Radiometry	Photometry
Energy	Radiant Energy	Luminous Energy
Flux (Power)	Radiant Power	Luminous Power
Flux Density	Irradiance (incoming) Radiosity (outgoing)	Illuminance (incoming) Luminosity (outgoing)
Angular Flux Density	Radiance	Luminance
Intensity	Radiant Intensity	Luminous Intensity

Radiometry versus photometry

Photometry	MKS	CGS	British
Luminous Energy	Talbot	Talbot	Talbot
Luminous Power	Lumen	Lumen	Lumen
Illuminance Luminosity	Lux	Phot	Footcandle
Luminance	Nit, Apostlib, Blondel	Stilb Lambert	Footlambert
Luminous Intensity	Candela	Candela	Candela

Modern LED light

Input power: 11 W

Output: 815 lumens

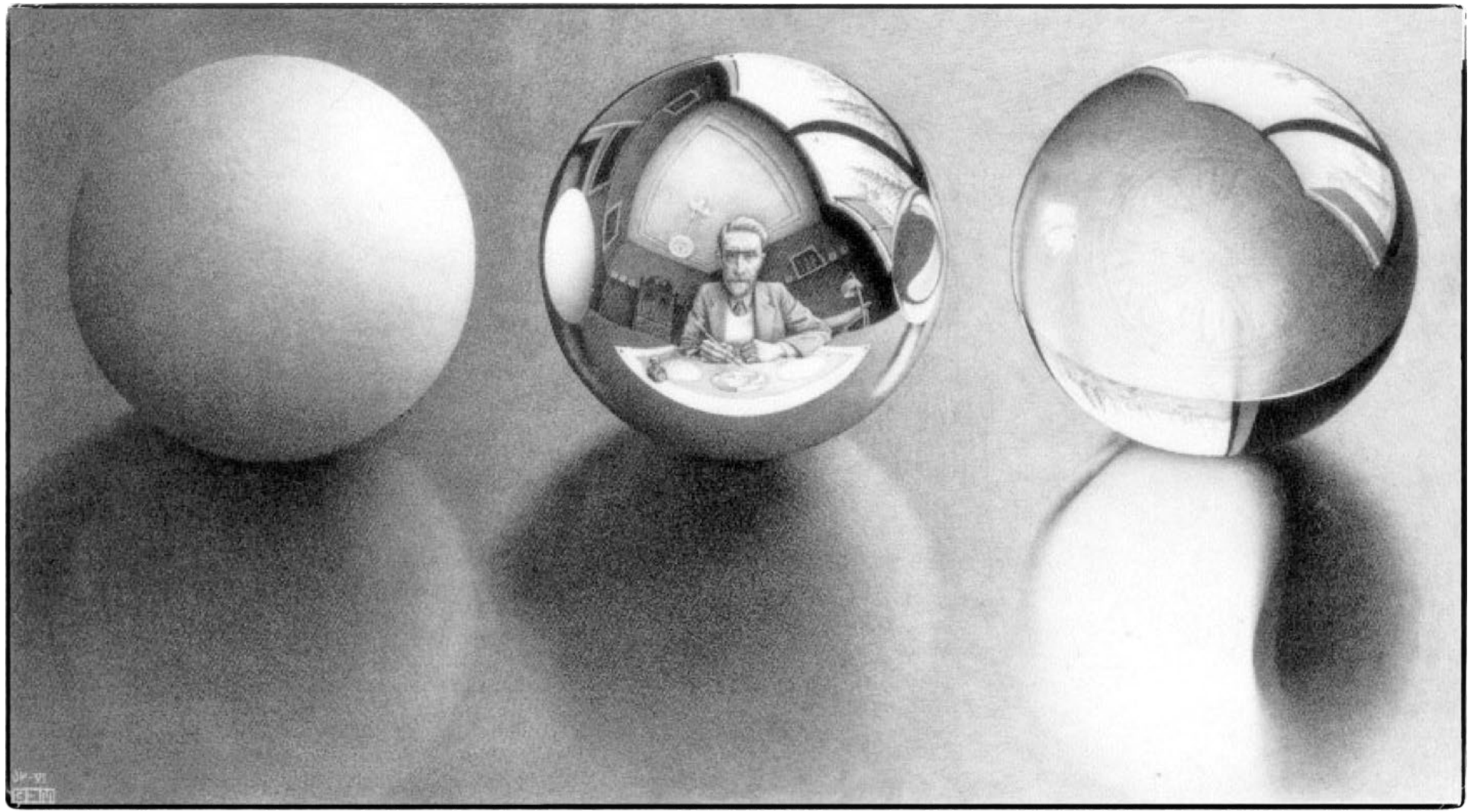
(~ 80 lumens / Watt)

Incandescent bulbs:

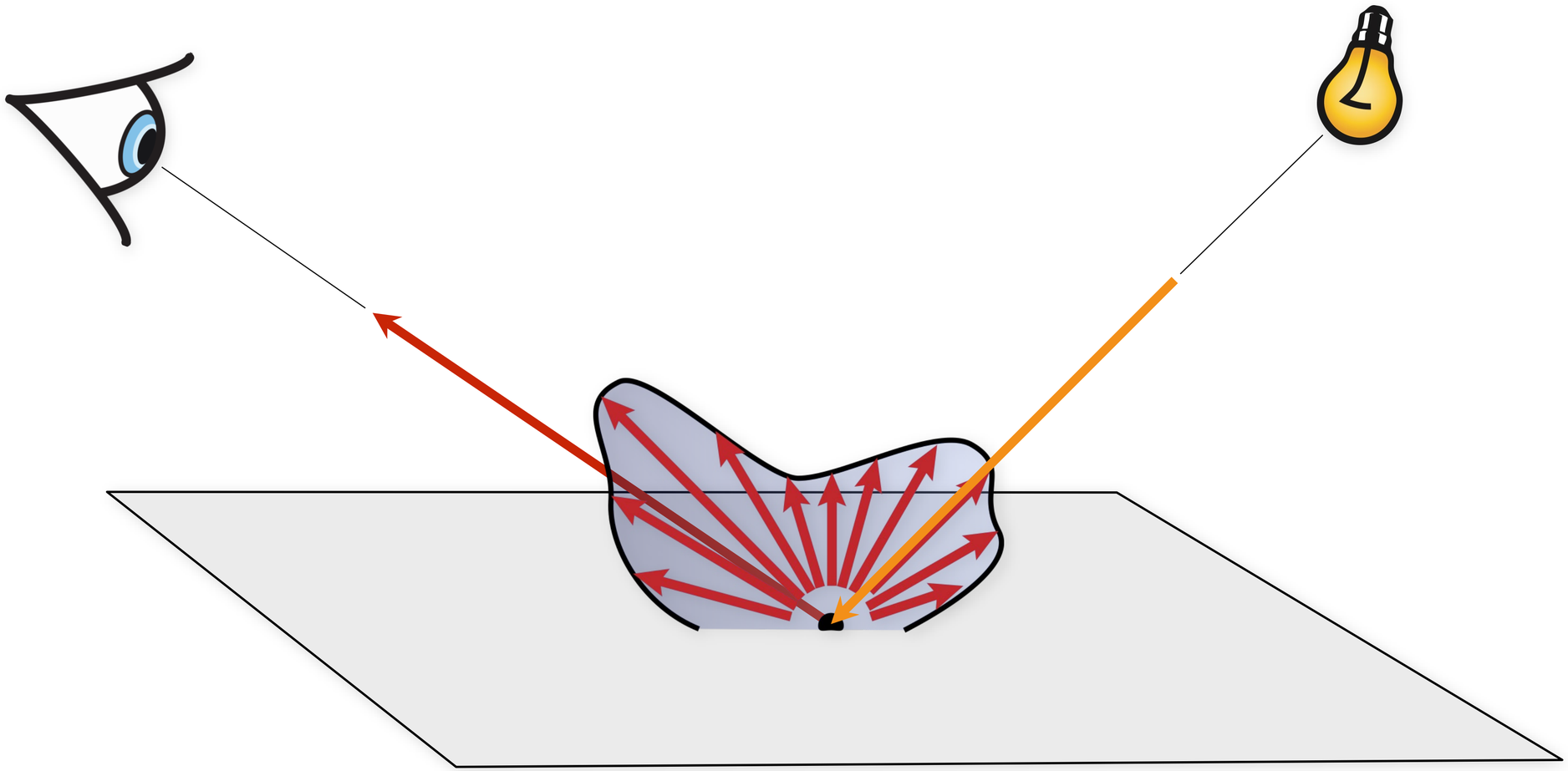
~15 lumens / Watt)



Reflection equation



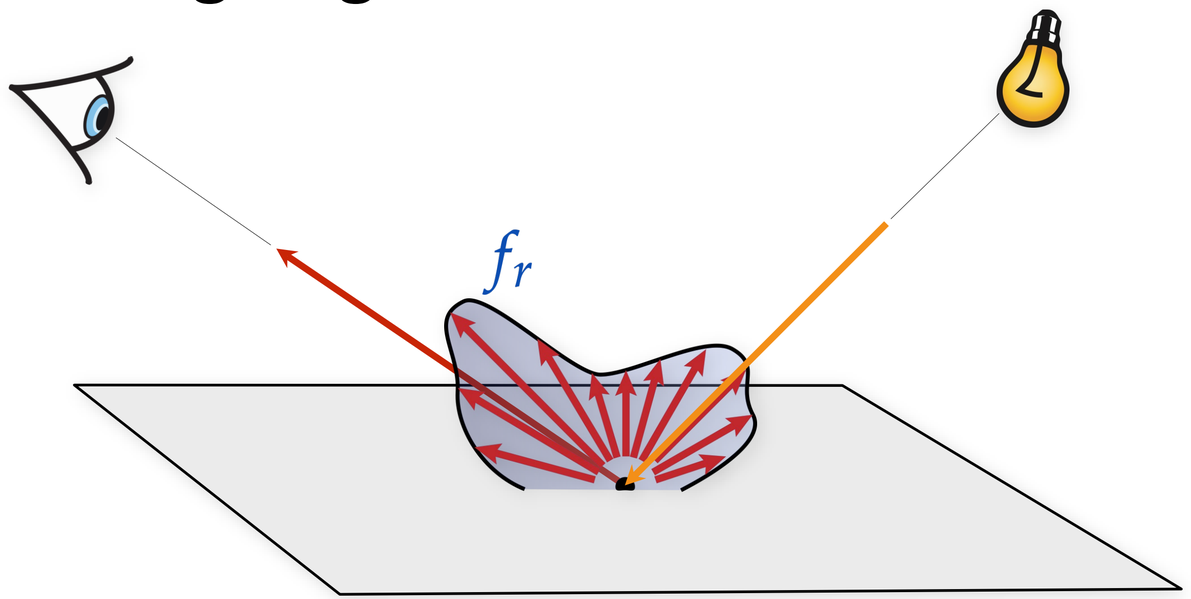
Light-Material Interactions



The BRDF

Bidirectional **R**eflectance **D**istribution **F**unction

- how much light gets scattered from one direction into each other direction
- formally: ratio of outgoing *radiance* to incident *irradiance*

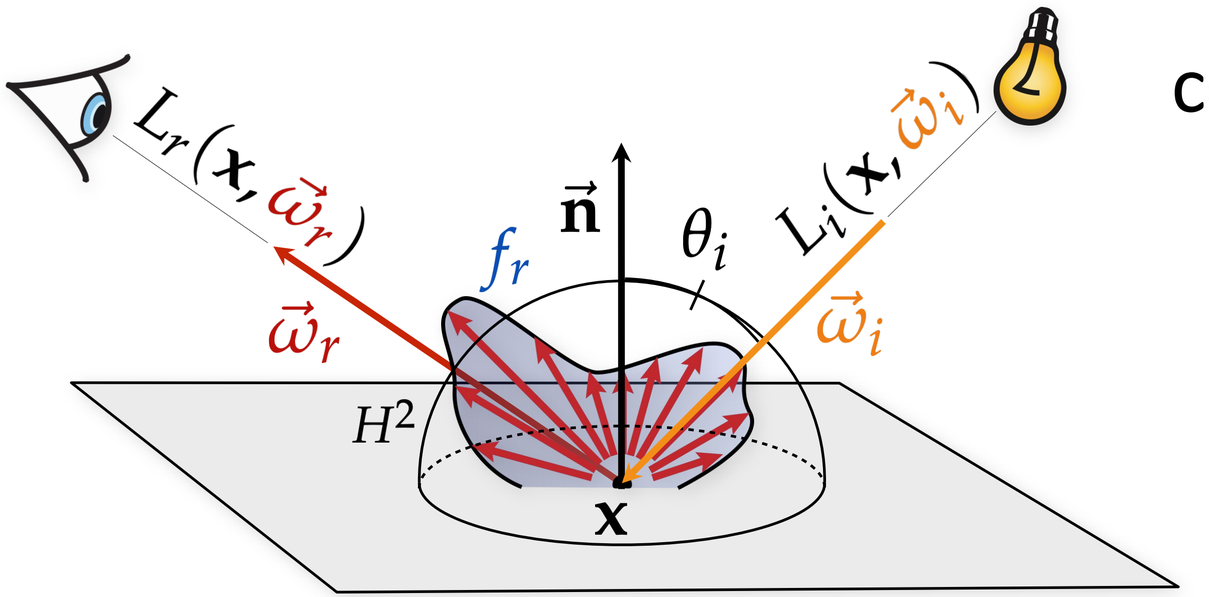


The reflection equation

Reflected radiance is a (hemi)spherical integral of incident radiance from all directions

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, \mathrm{d}\vec{\omega}_i$$

Where does the cosine come from?



This describes a local illumination model

Motivation



Motivation



BRDF Properties

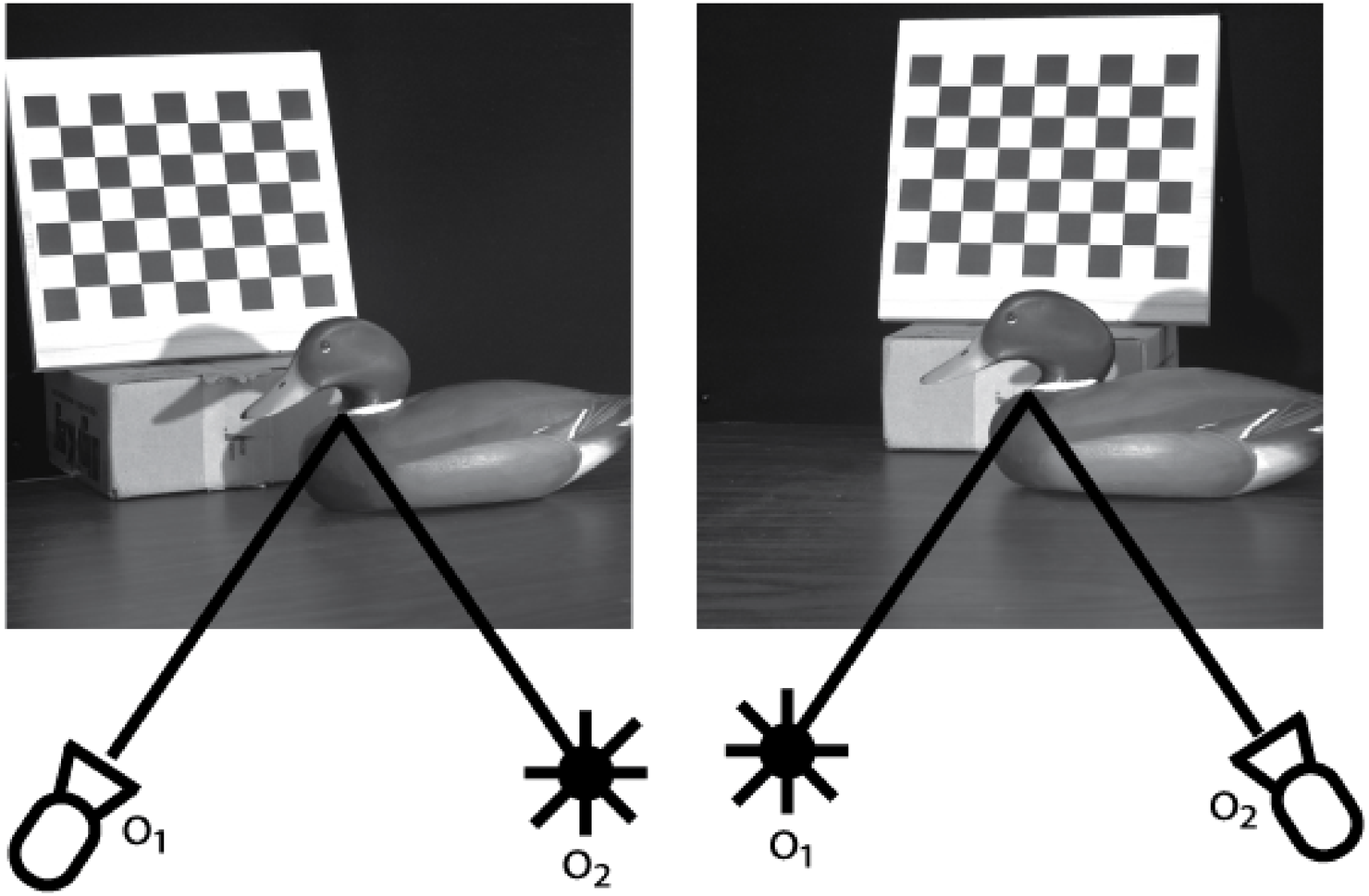
Real/physically-plausible BRDFs obey:

- Energy conservation

$$\int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) \cos\theta_i \, \mathrm{d}\vec{\omega}_i \leq 1, \quad \forall \vec{\omega}_r$$

Where does the cosine come from?

Helmholtz Reciprocity



BRDFs Properties

Real/physically-plausible BRDFs obey:

- Energy conservation

$$\int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) \cos\theta_i \, \mathrm{d}\vec{\omega}_i \leq 1, \quad \forall \vec{\omega}_r$$

- Helmholtz reciprocity

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = f_r(\mathbf{x}, \vec{\omega}_r, \vec{\omega}_i)$$

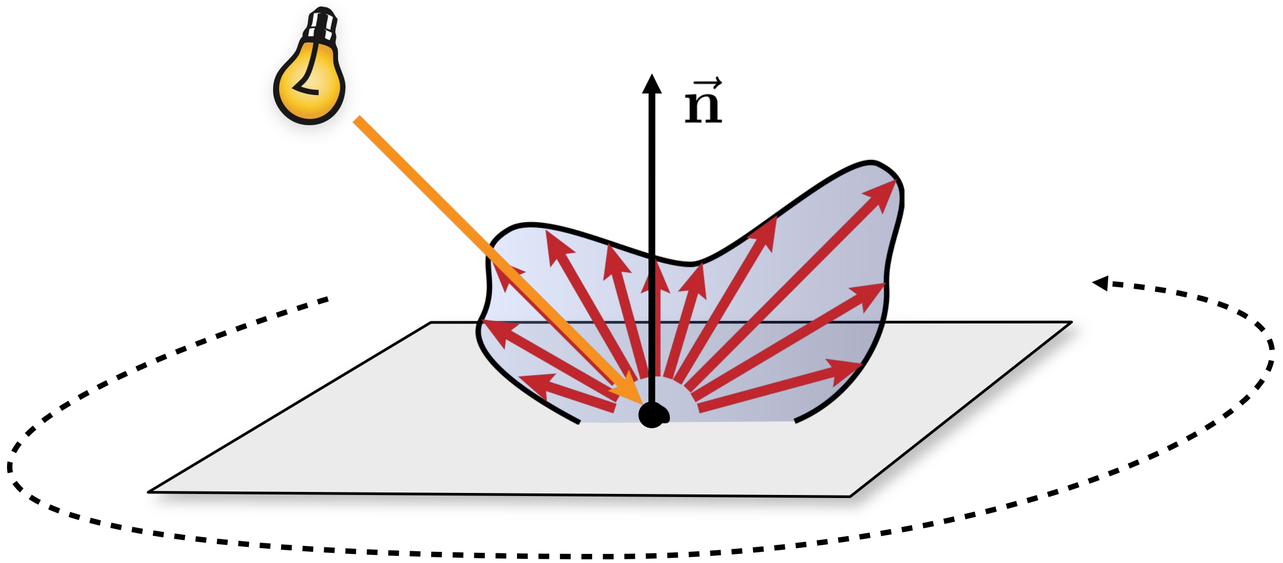
$$f_r(\mathbf{x}, \vec{\omega}_i \leftrightarrow \vec{\omega}_r)$$

BRDFs Properties

If the BRDF is unchanged as the material is rotated around the normal, then it is ***isotropic***, otherwise it is ***anisotropic***.

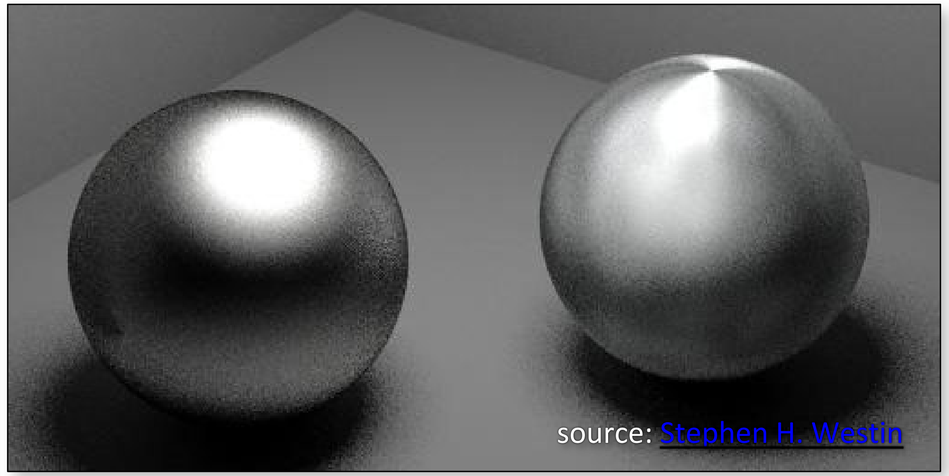
Isotropic BRDFs are functions of just 3 variables

$$(\theta_i, \theta_r, \Delta\phi)$$



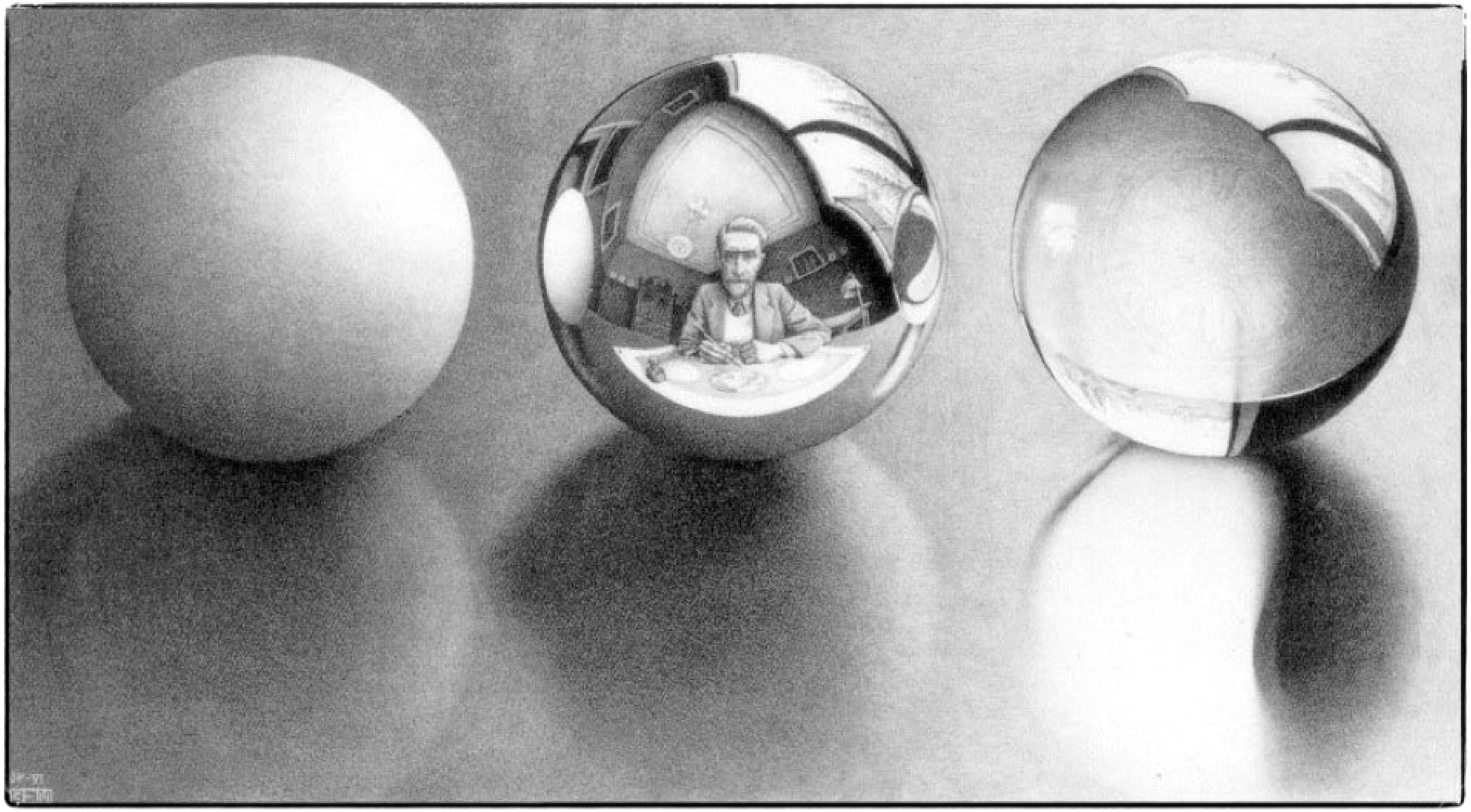
Isotropic vs Anisotropic Reflection



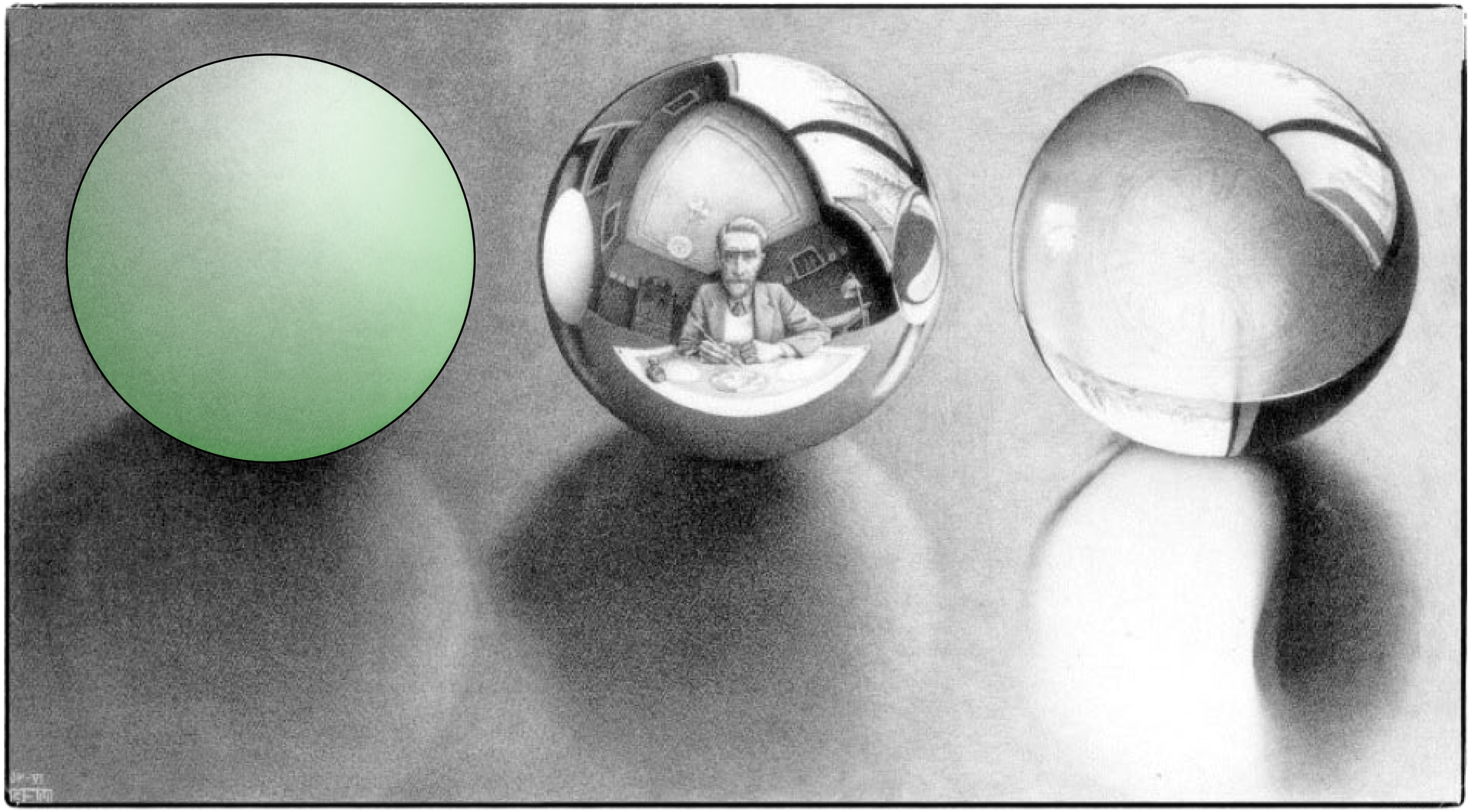




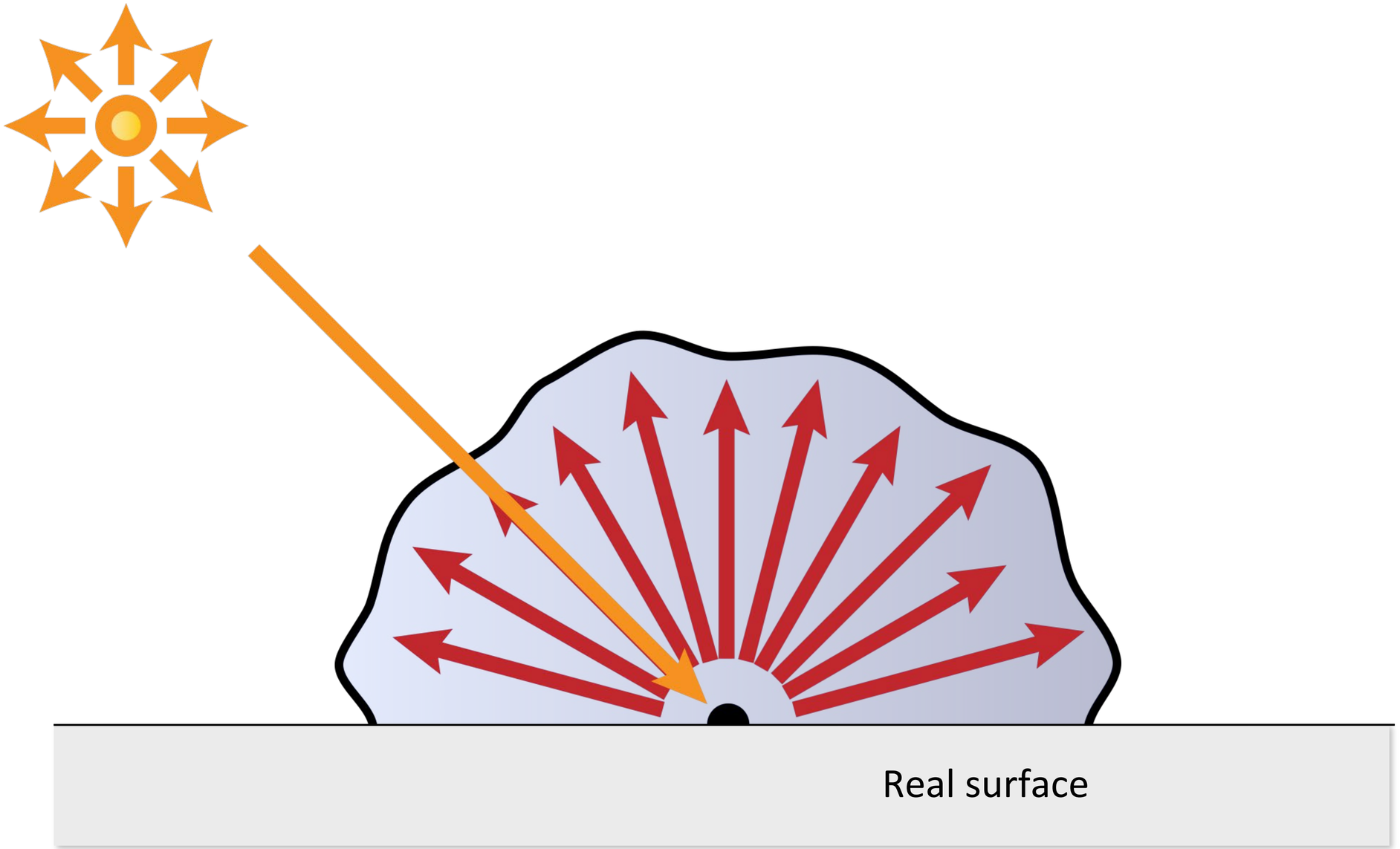
Idealized materials



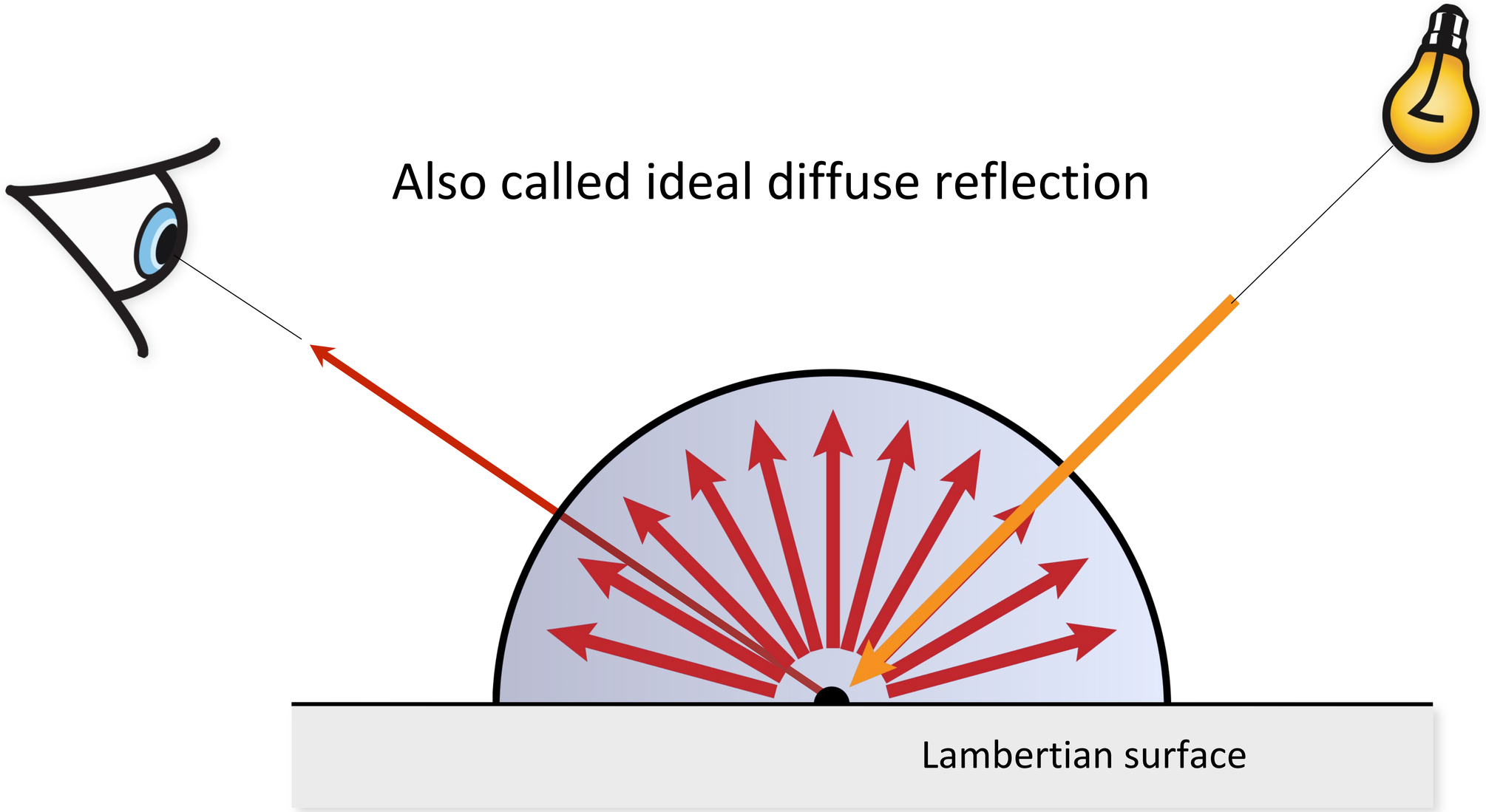
Diffuse reflection



Diffuse reflection



Lambertian reflection



BRDF for ideal diffuse reflection?

Reflected radiance is a (hemi)spherical integral of incident radiance from all directions

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, \mathrm{d}\vec{\omega}_i$$

Scatters light equal in all directions
BRDF is a constant

Ideal Diffuse BRDF

For Lambertian reflection, the BRDF is a constant:

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

Note: we can drop ω_r

$$L_r(\mathbf{x}) = f_r \int_{H^2} L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

$$L_r(\mathbf{x}) = f_r \, E(\mathbf{x})$$

If *all* incoming light is reflected:

$$E(\mathbf{x}) = B(\mathbf{x})$$

$$E(\mathbf{x}) = \int_{H^2} L_r(\mathbf{x}) \cos\theta \, d\vec{\omega}$$

Note: can also be derived from energy conservation

$$E(\mathbf{x}) = L_r(\mathbf{x}) \int_{H^2} \cos\theta \, d\vec{\omega}$$

$$E(\mathbf{x}) = L_r(\mathbf{x}) \, \pi$$

$$f_r = \frac{1}{\pi}$$

Diffuse BRDF

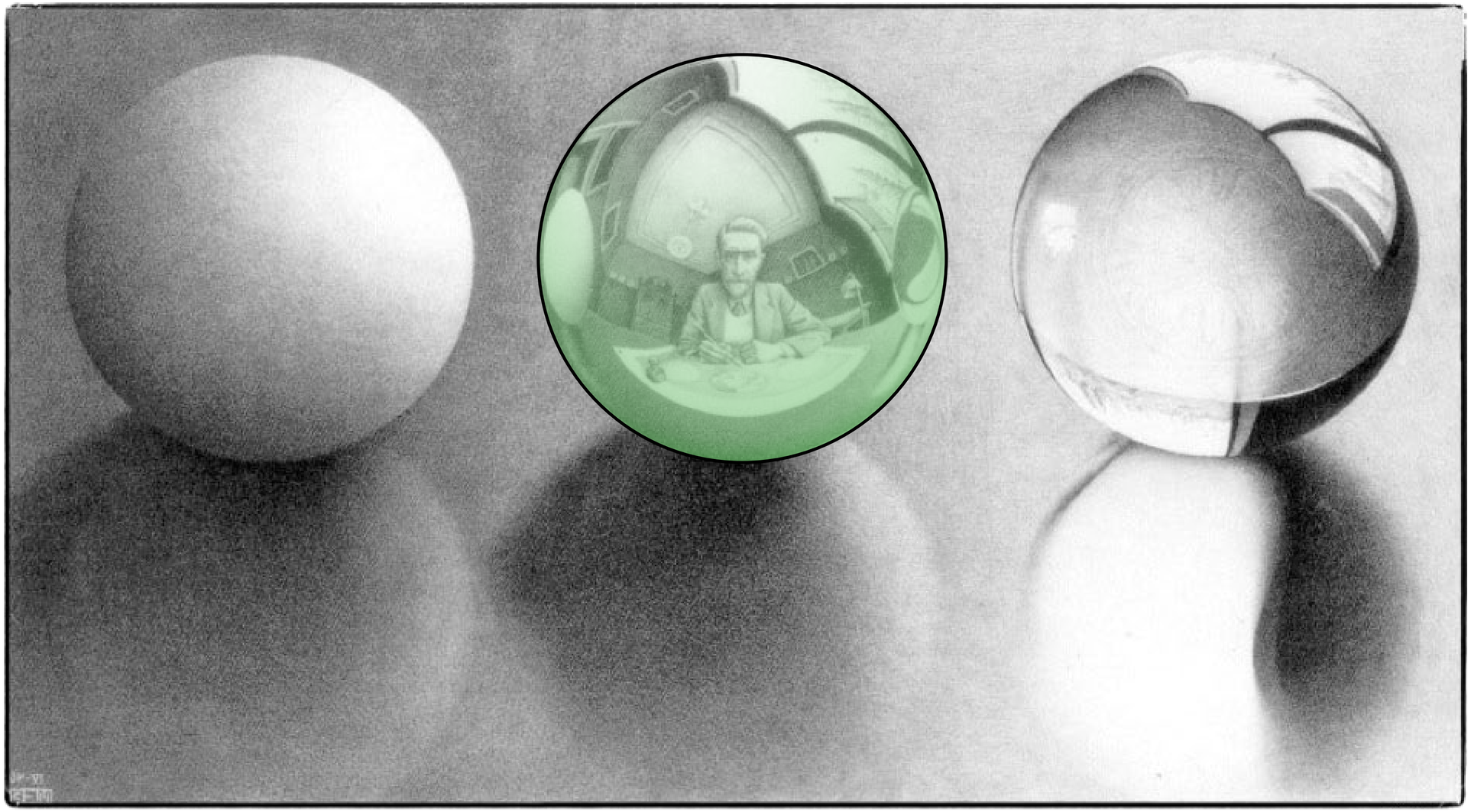
For Lambertian reflection, the BRDF is a constant:

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

$$L_r(\mathbf{x}) = \frac{\rho}{\pi} \int_{H^2} L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

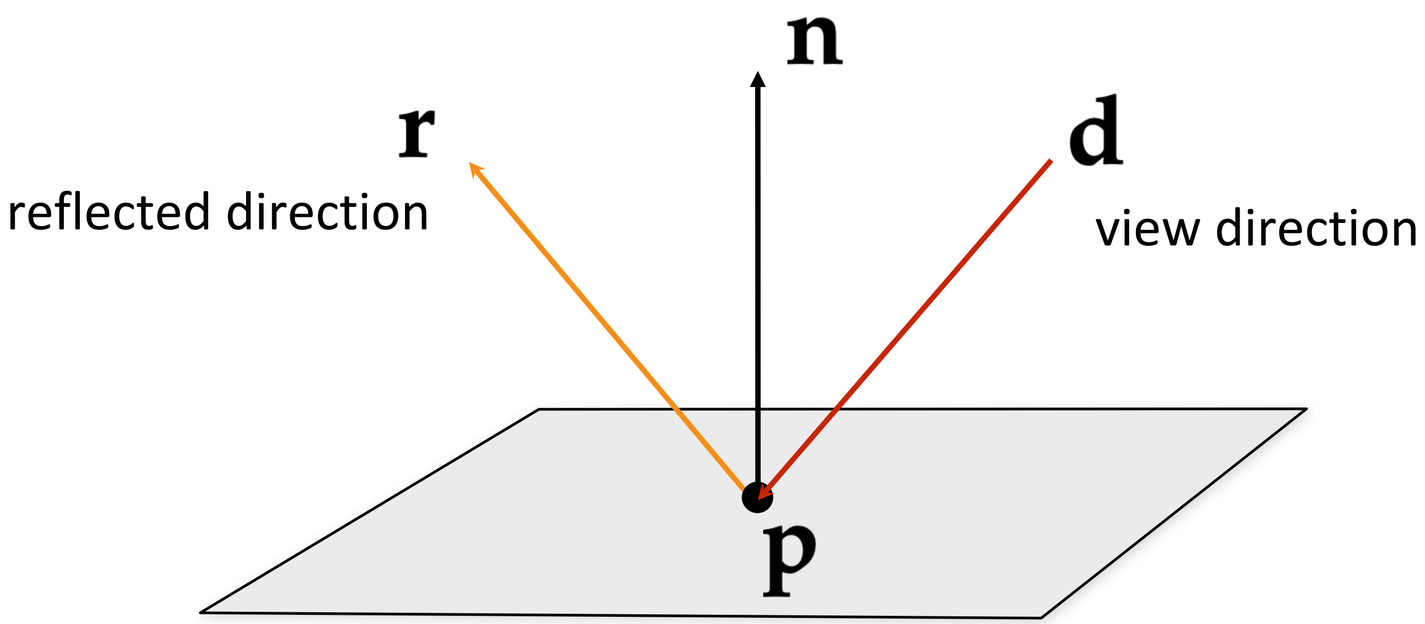
ρ: Diffuse reflectance (albedo) [0…1)

Specular/Mirror reflection



Mirror reflection

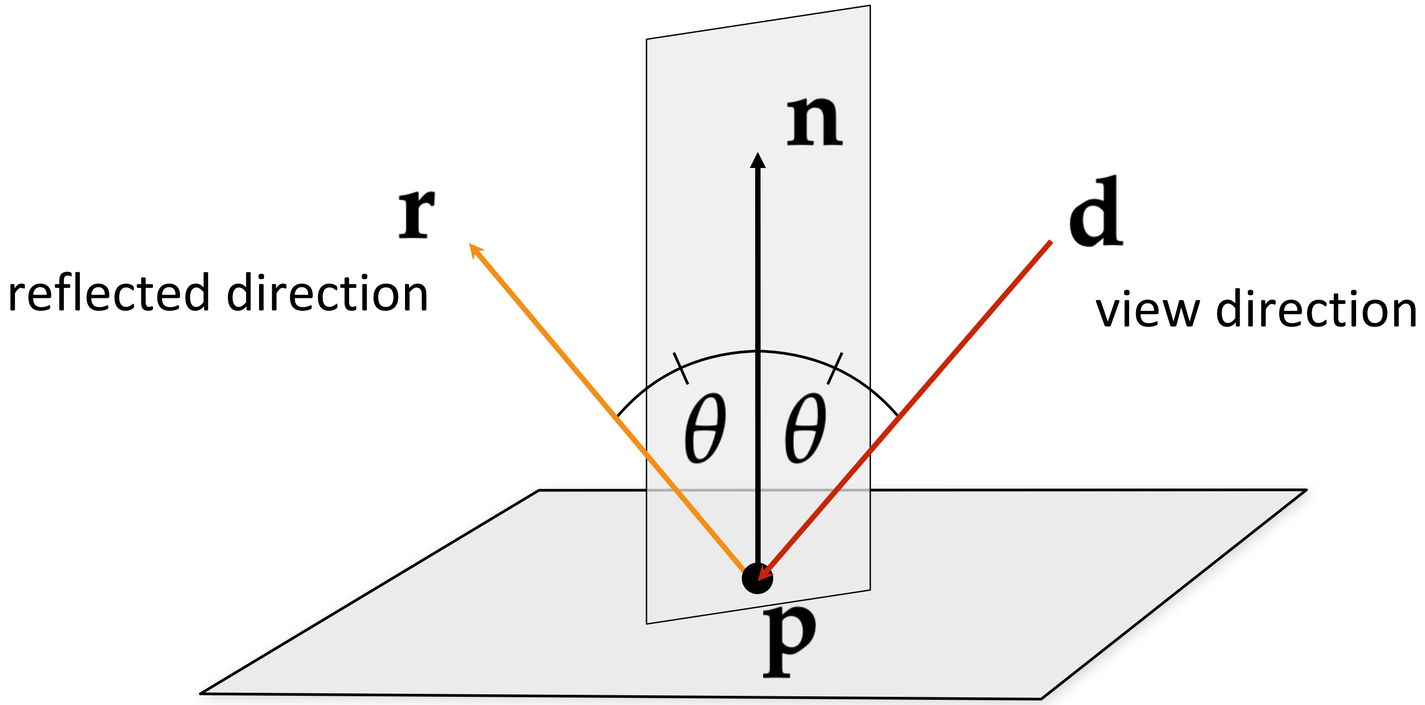
Assume $\mathbf{n}$ is unit length



What <u>two</u> properties defined reflection direction?

Mirror reflection

Assume $\mathbf{n}$ is unit length

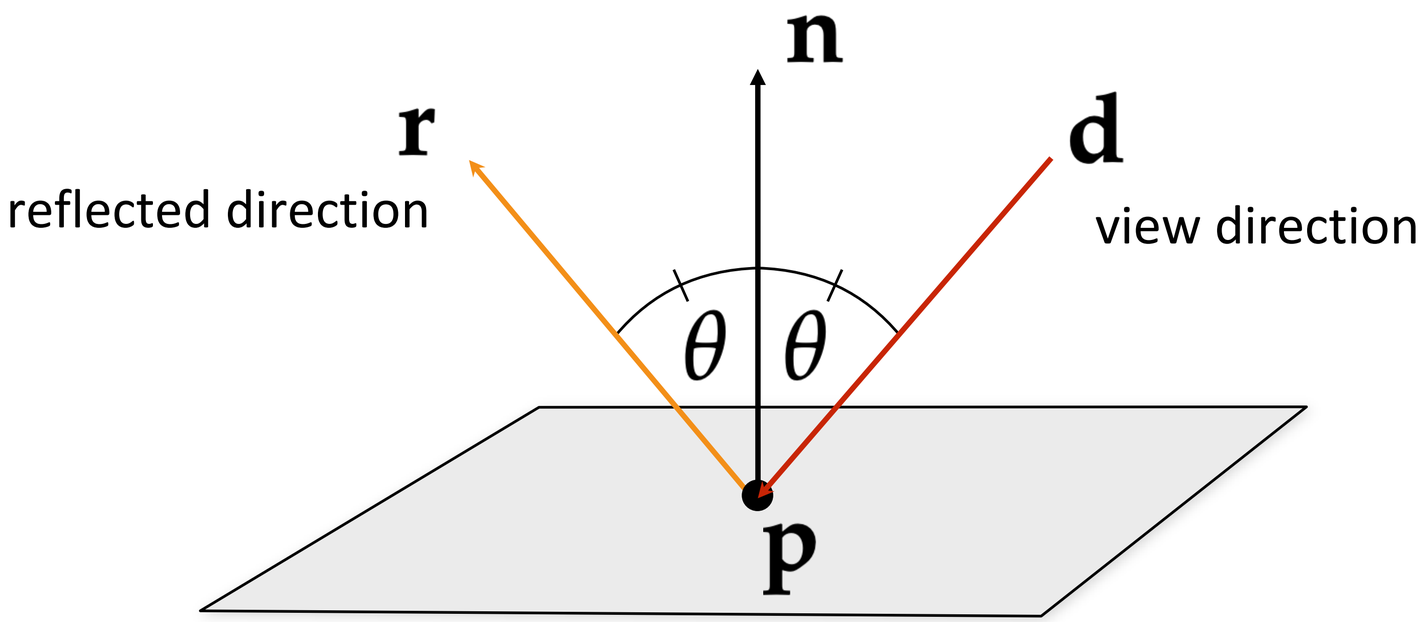


What <u>two</u> properties defined reflection direction?

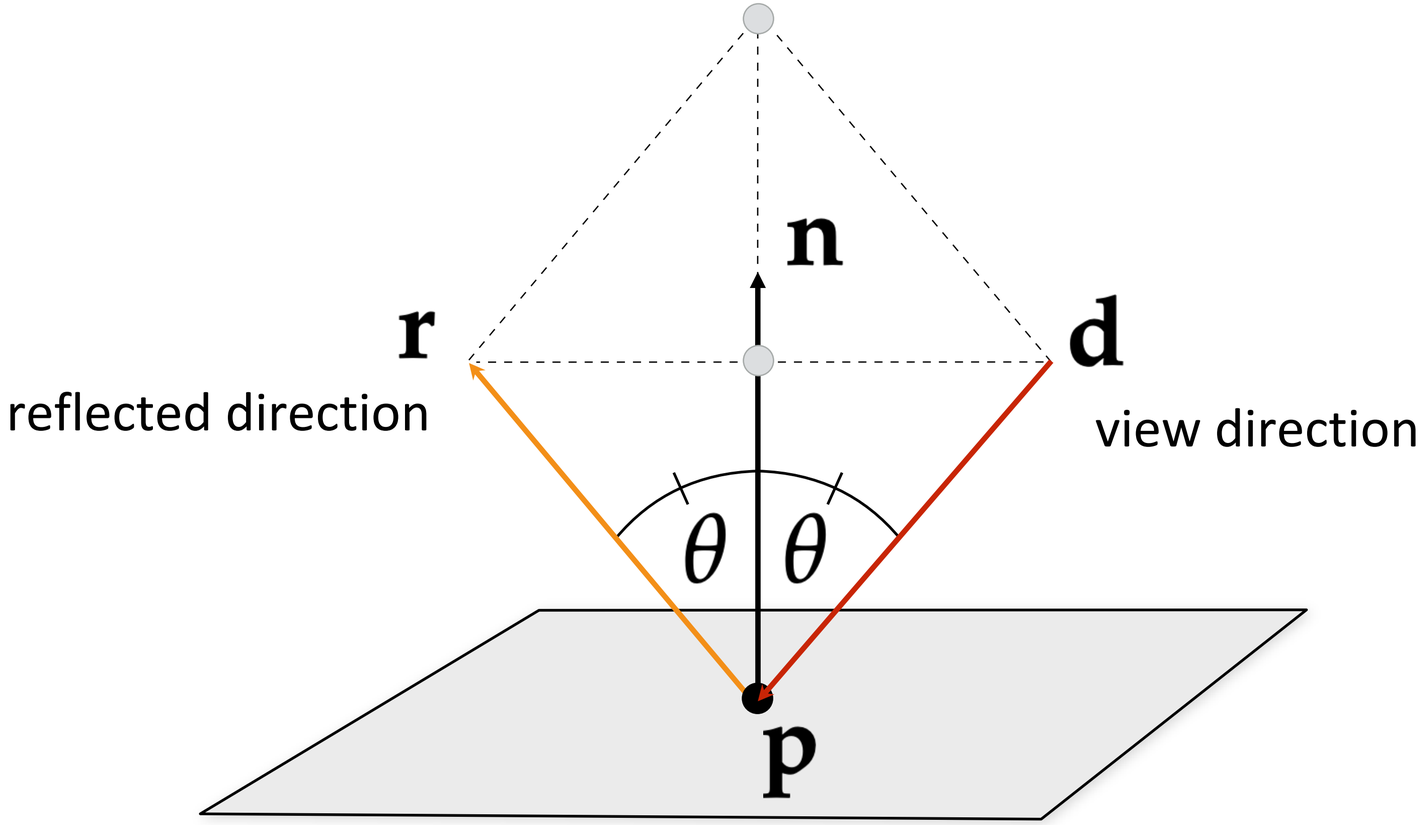
- co-planar view direction, reflected direction, and normal direction
- equal angles between normal-view directions, and normal-reflected directions

Mirror reflection

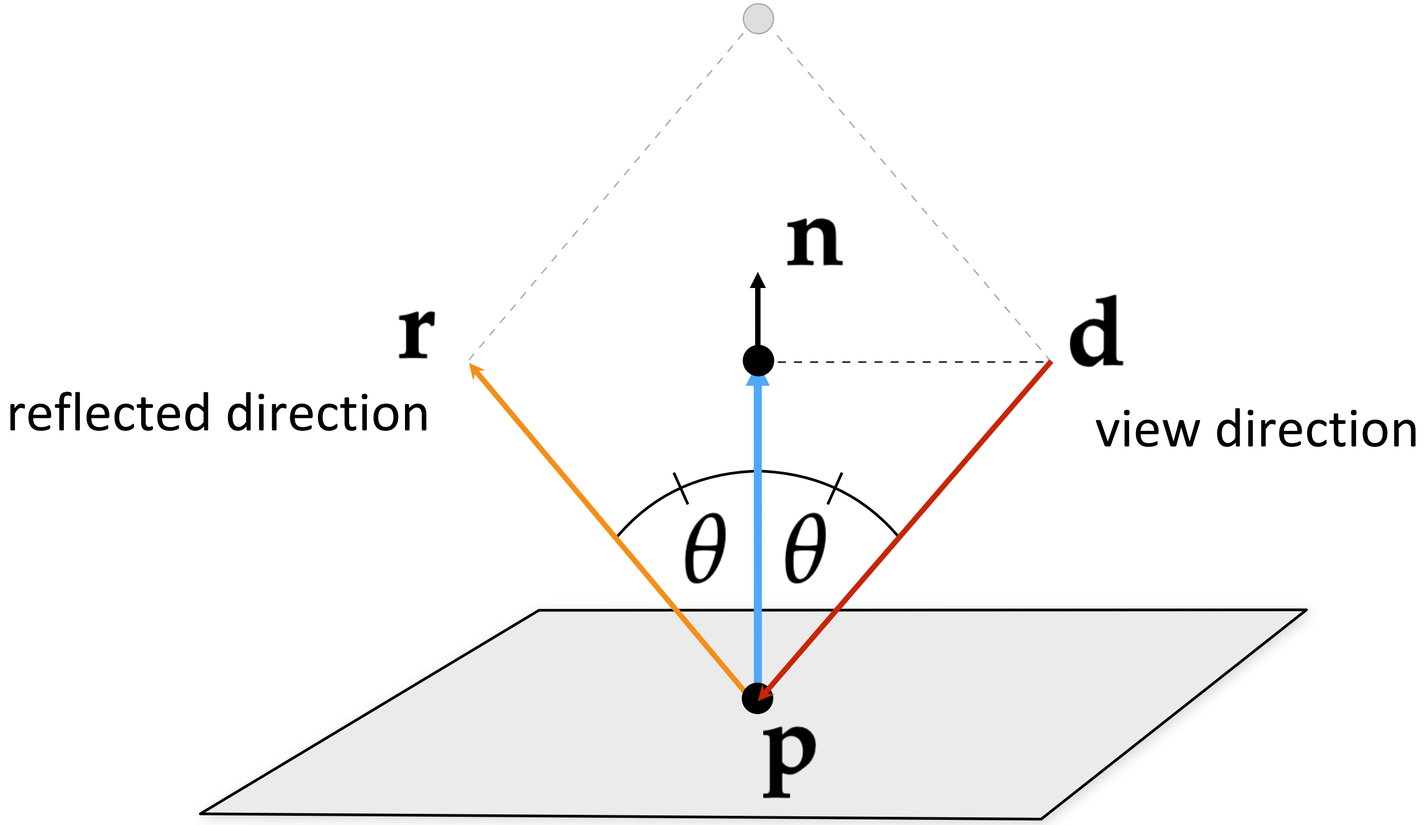
Assume $\mathbf{n}$ is unit length



Mirror reflection

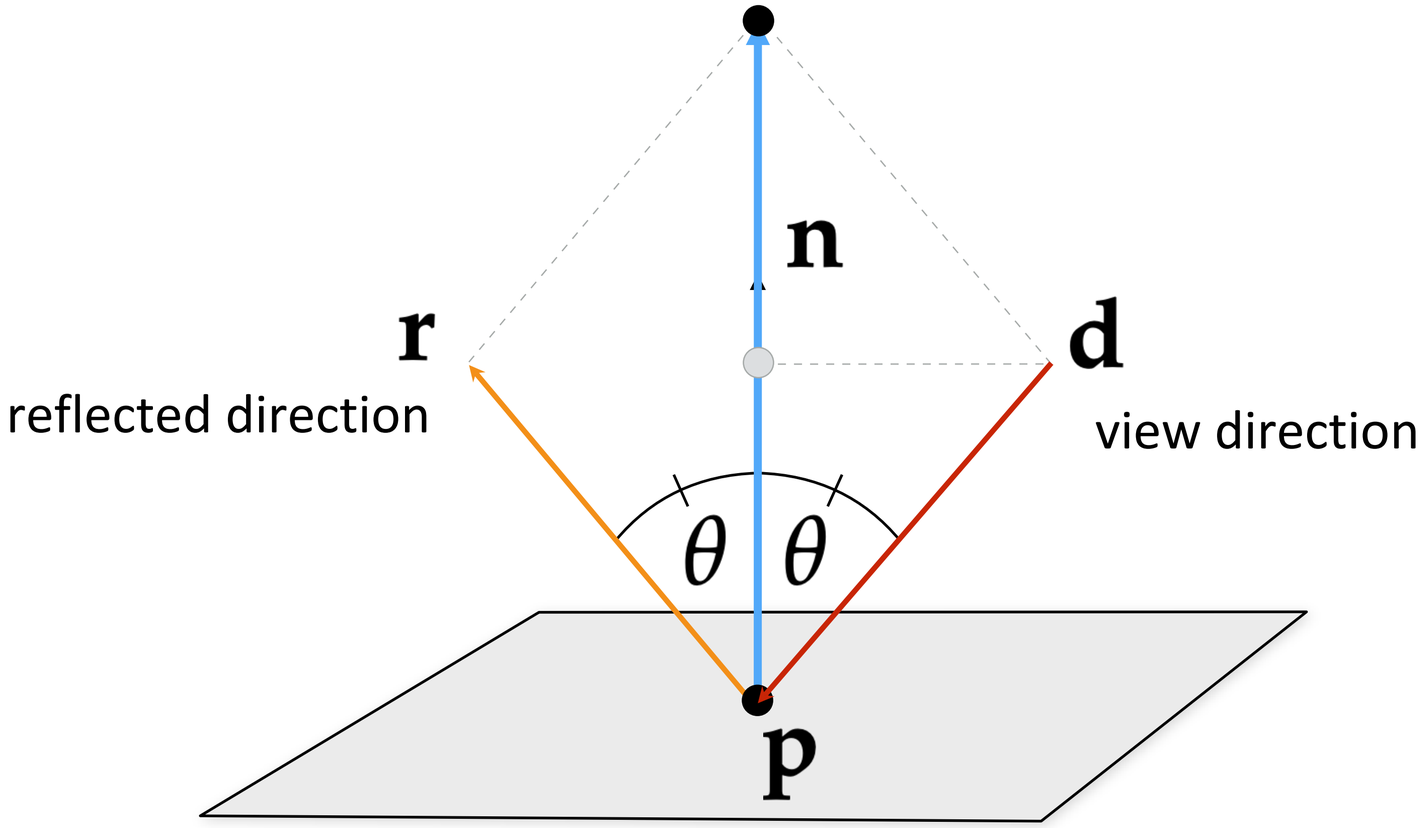


Mirror reflection



$$-\mathbf{n}(\mathbf{n}\cdot\mathbf{d})$$

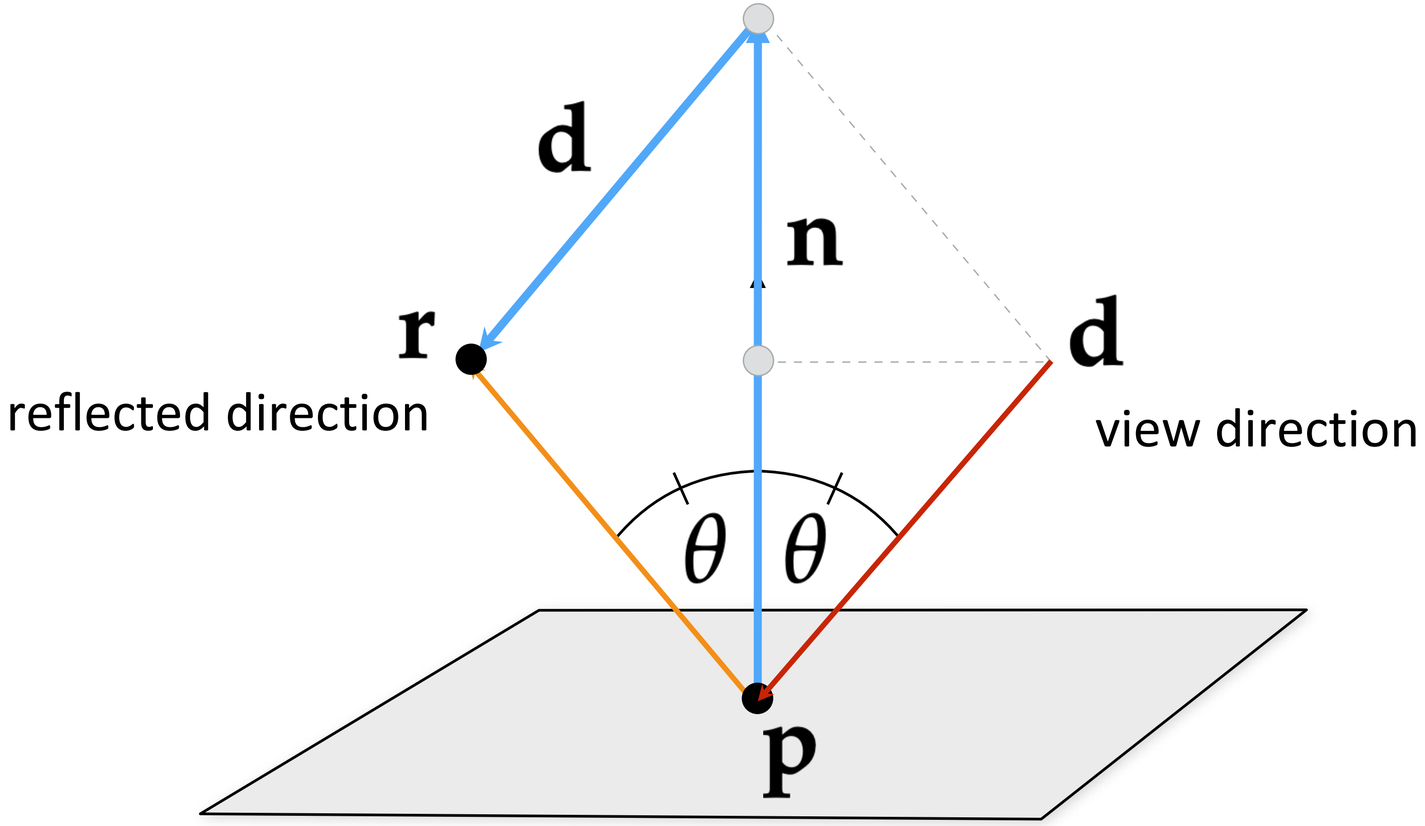
Mirror reflection



$$-2\mathbf{n}(\mathbf{n}\cdot\mathbf{d})$$

Mirror reflection

Assumes $\mathbf{n}$ is unit length



$$\mathbf{r} = -2\mathbf{n}(\mathbf{n} \cdot \mathbf{d}) + \mathbf{d}$$

Specular BRDF?

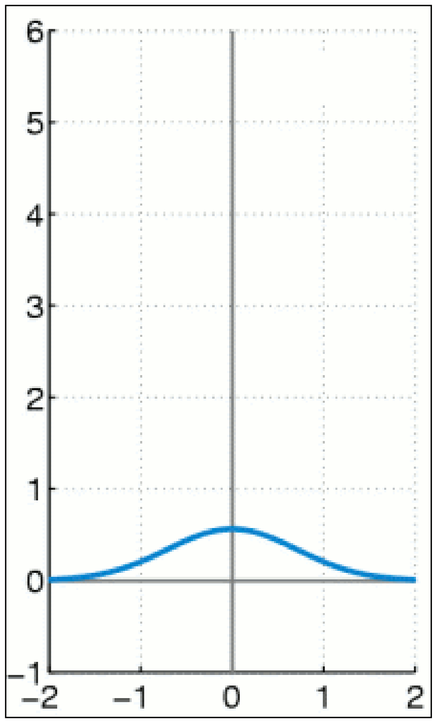
Reflected radiance is a (hemi)spherical integral of incident radiance from all directions

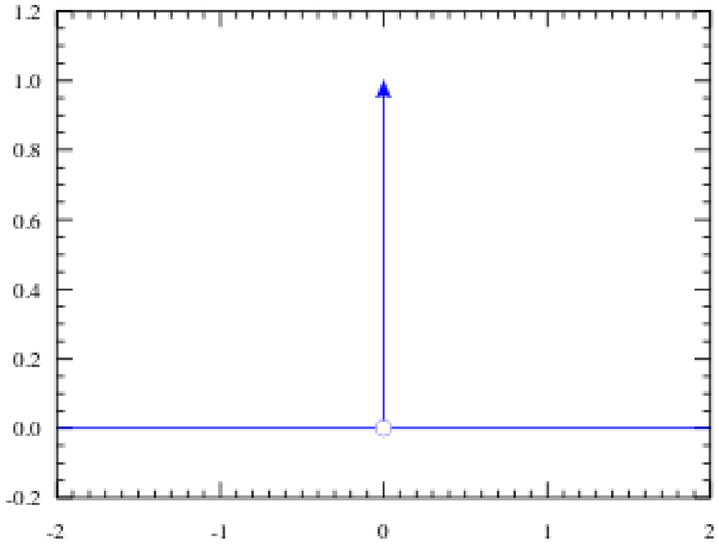
$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, \mathrm{d}\vec{\omega}_i$$

Scatters all light into one (or two) directions
Contains a Dirac delta
Integral drops out

What is the BRDF for specular reflection/refraction?

Dirac delta functions





$$\int_{-\infty}^{\infty} f(x)\delta(x-a)\,\mathrm{d}x = f(a)$$

Note: careful when performing changes of variables in Dirac delta functions!

BRDF of Ideal Specular Reflection

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, \mathrm{d}\vec{\omega}_i$$

What is the BRDF for specular reflection?

Dirac delta

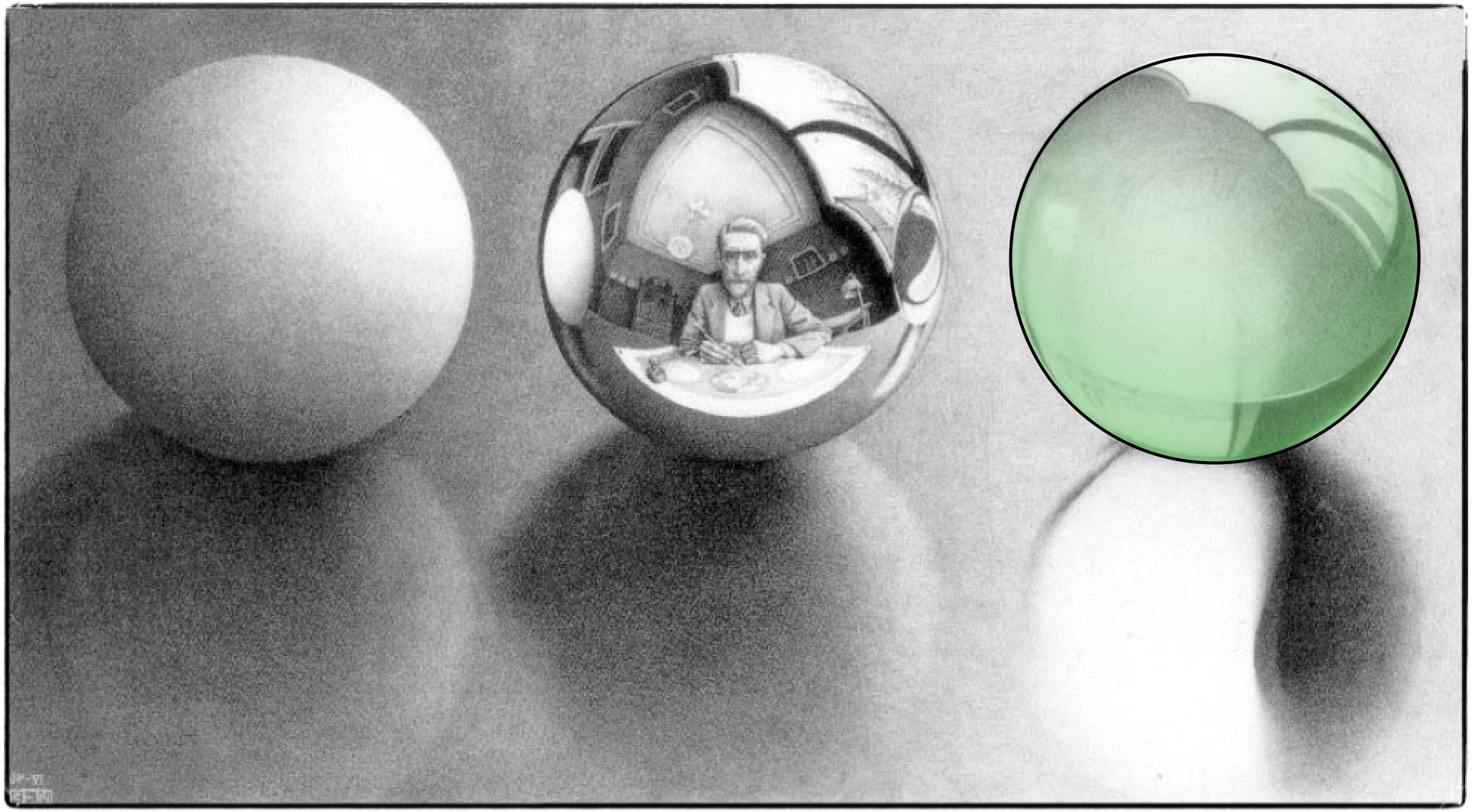
Fresnel reflection

Reflection function
(flips about normal)

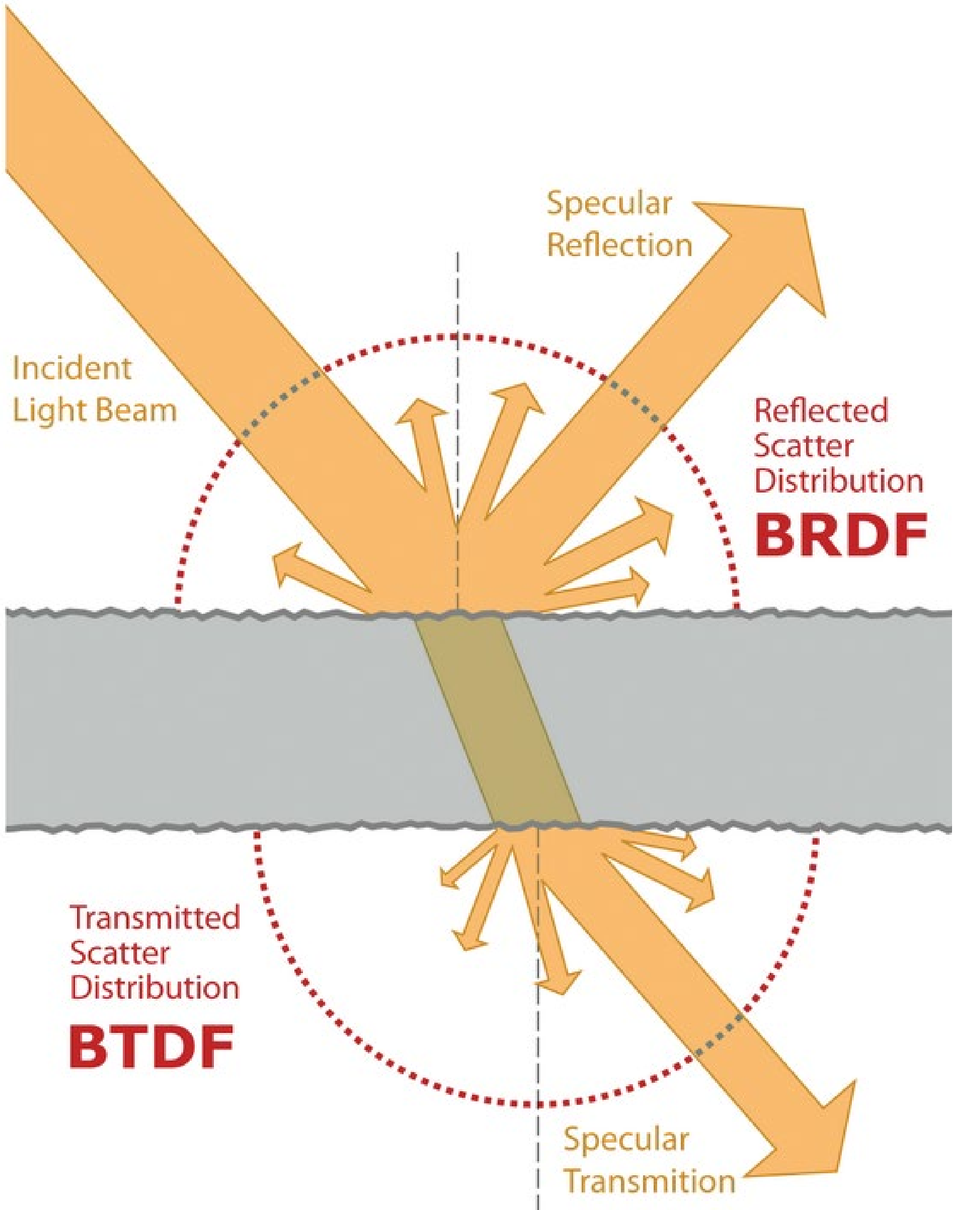
$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = F_r(\vec{\omega}_i) \frac{\delta(\vec{\omega}_i - R(\vec{\omega}_r, \vec{\mathbf{n}}))}{\cos\theta_i}$$

to cancel the cosine term
in the reflection equation
(Fresnel eqs. account for it)

Specular refraction



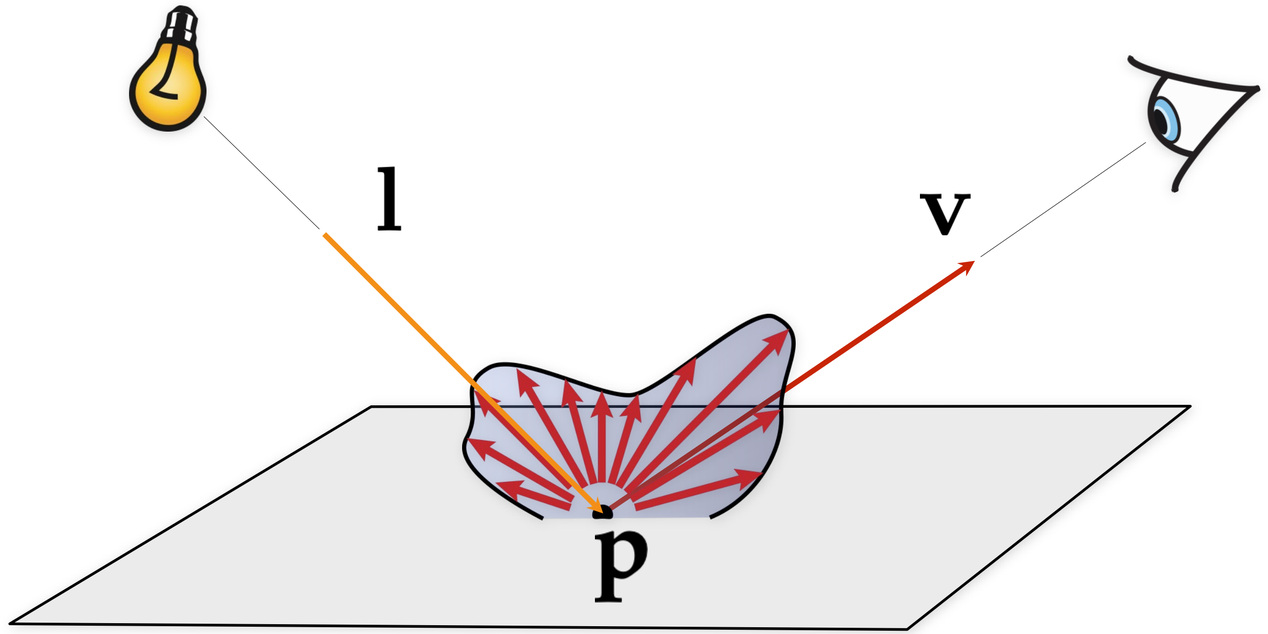
Reflection vs. Refraction



The BSDF

Bidirectional **S**cattering **D**istribution **F**unction

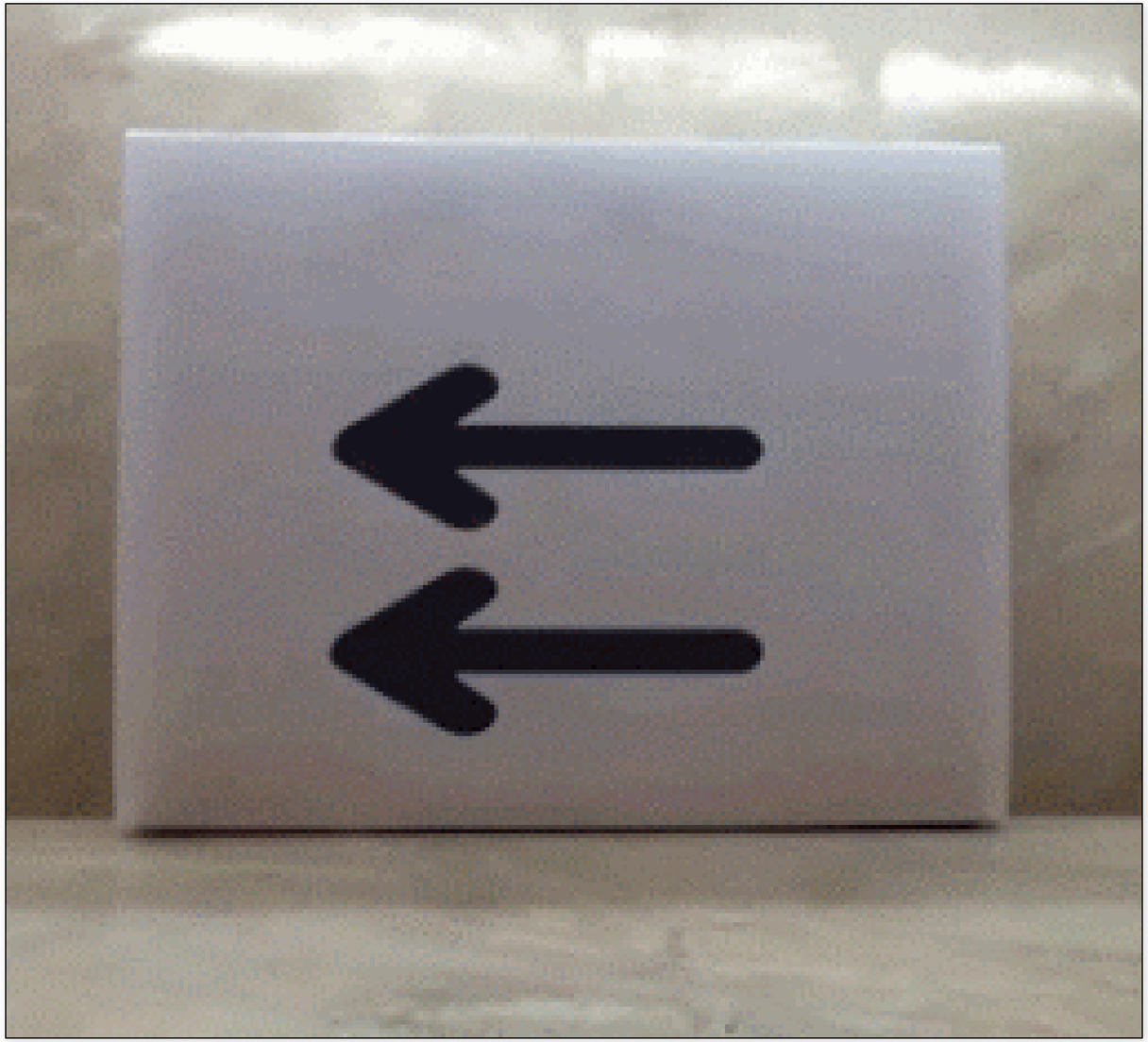
- informally: how much the material scatters light coming from one direction $\mathbf{l}$ into some other direction $\mathbf{v}$, at each point $\mathbf{p}$



Refraction



Refraction



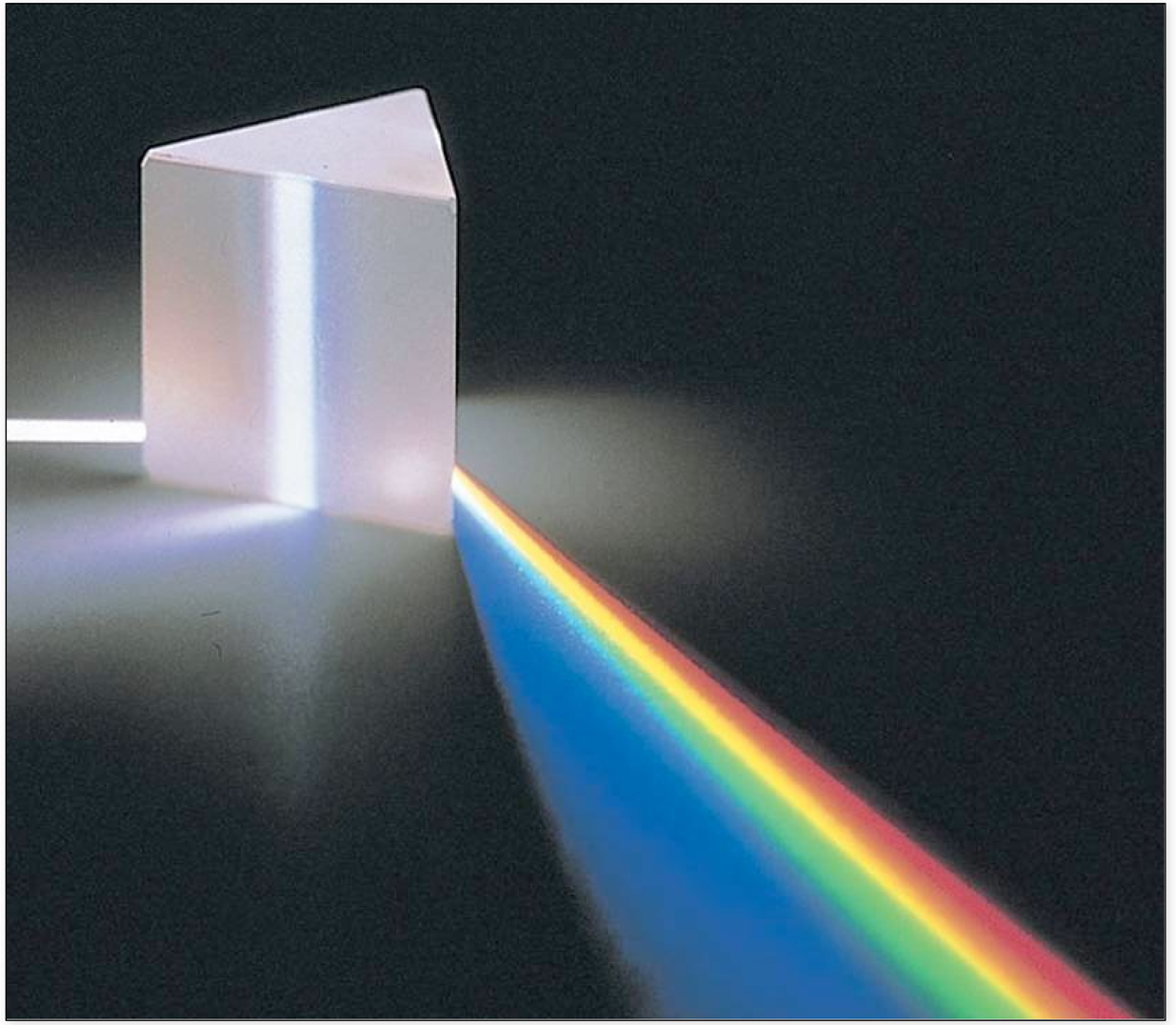
Index of Refraction

Speed of light in vacuum / speed of light in medium

Some values of η	
Vacuum	1
Air at STP	1.00029
Ice	1.31
Water	1.33
Crown glass	1.52 - 1.65
Diamond	2.417

These are actually wavelength dependent!

Dispersion



Double rainbow all the way across the sky!

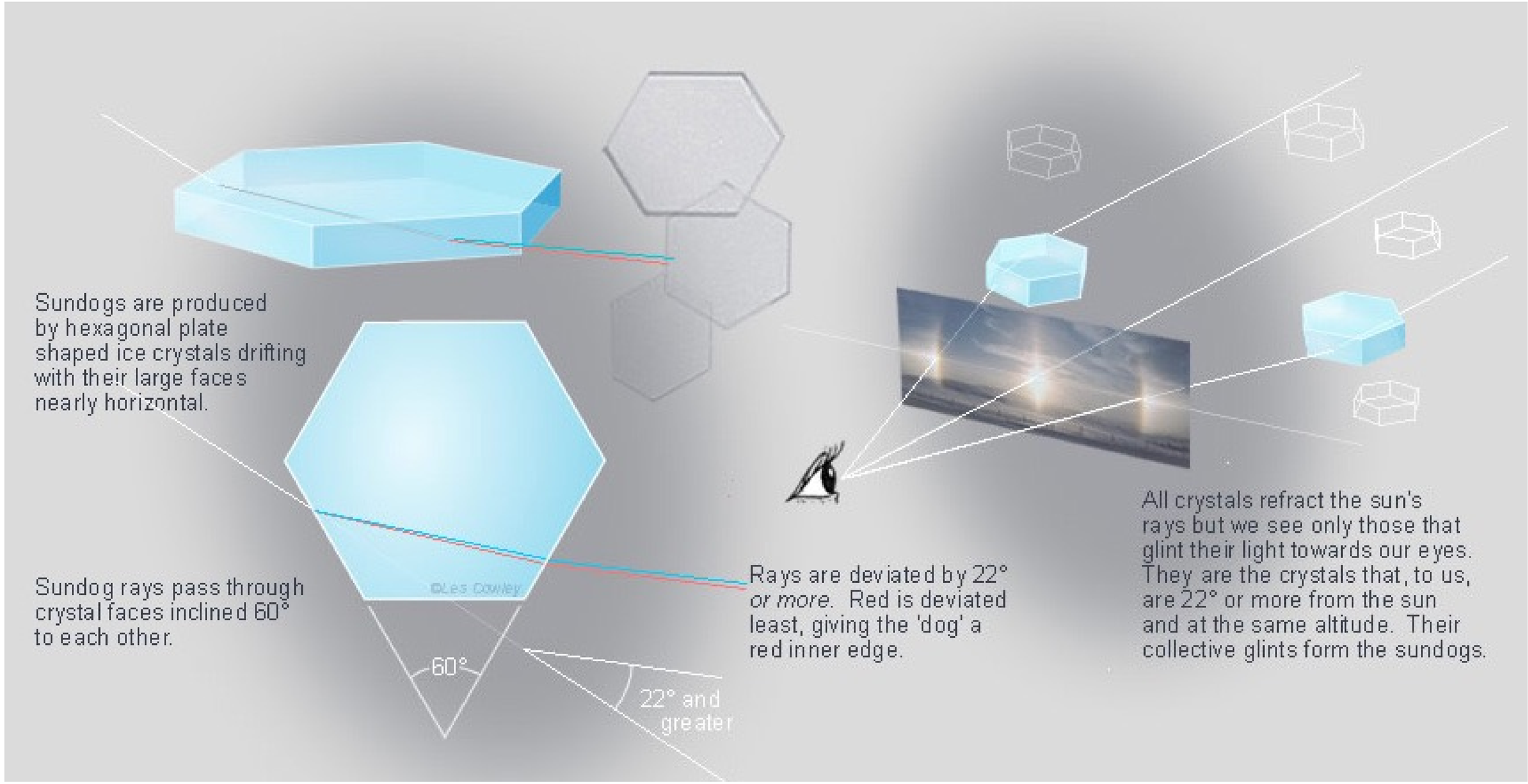


Dispersion: “Halos” and “Sun dogs”



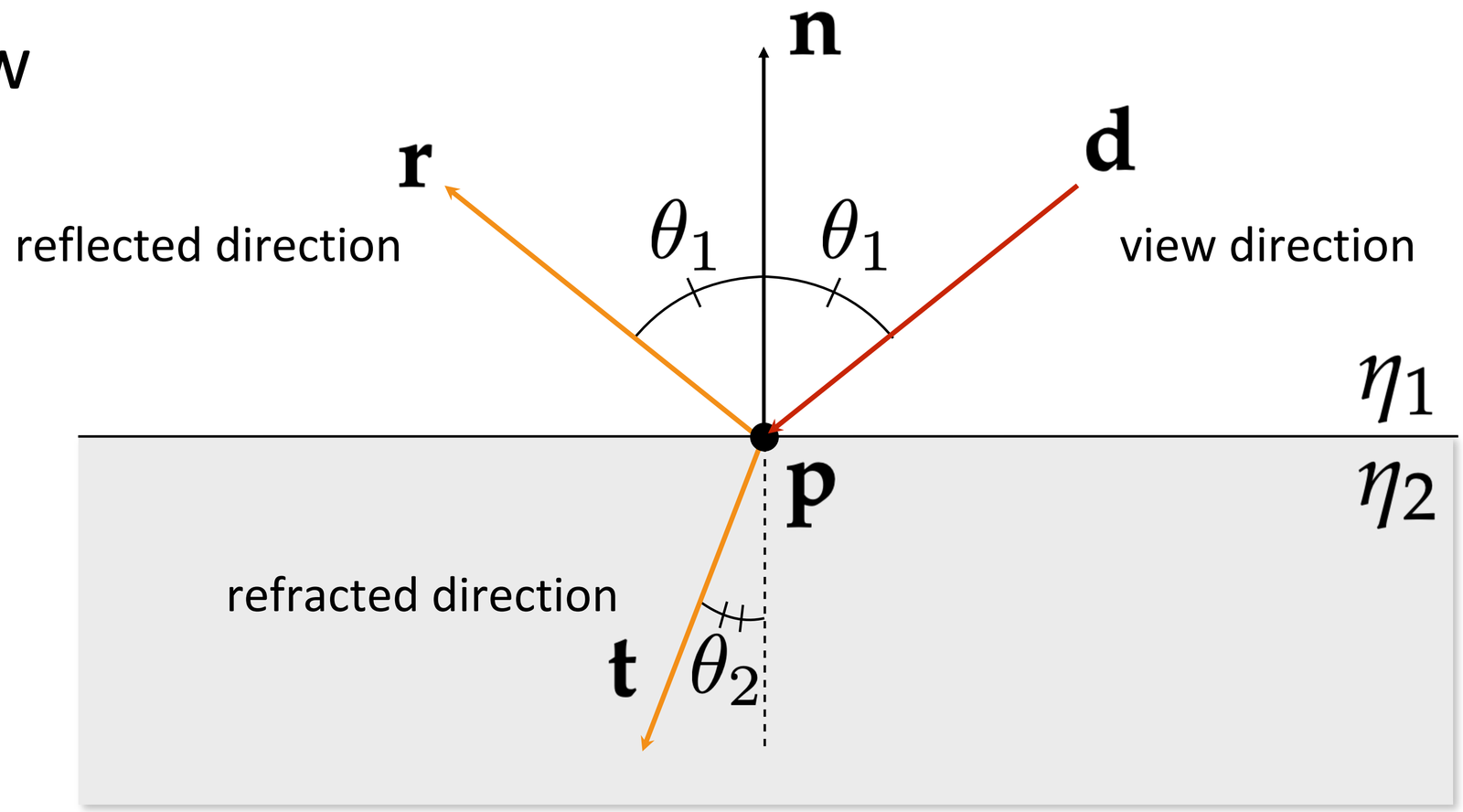


Halos and Sundogs



Specular transmission/refraction

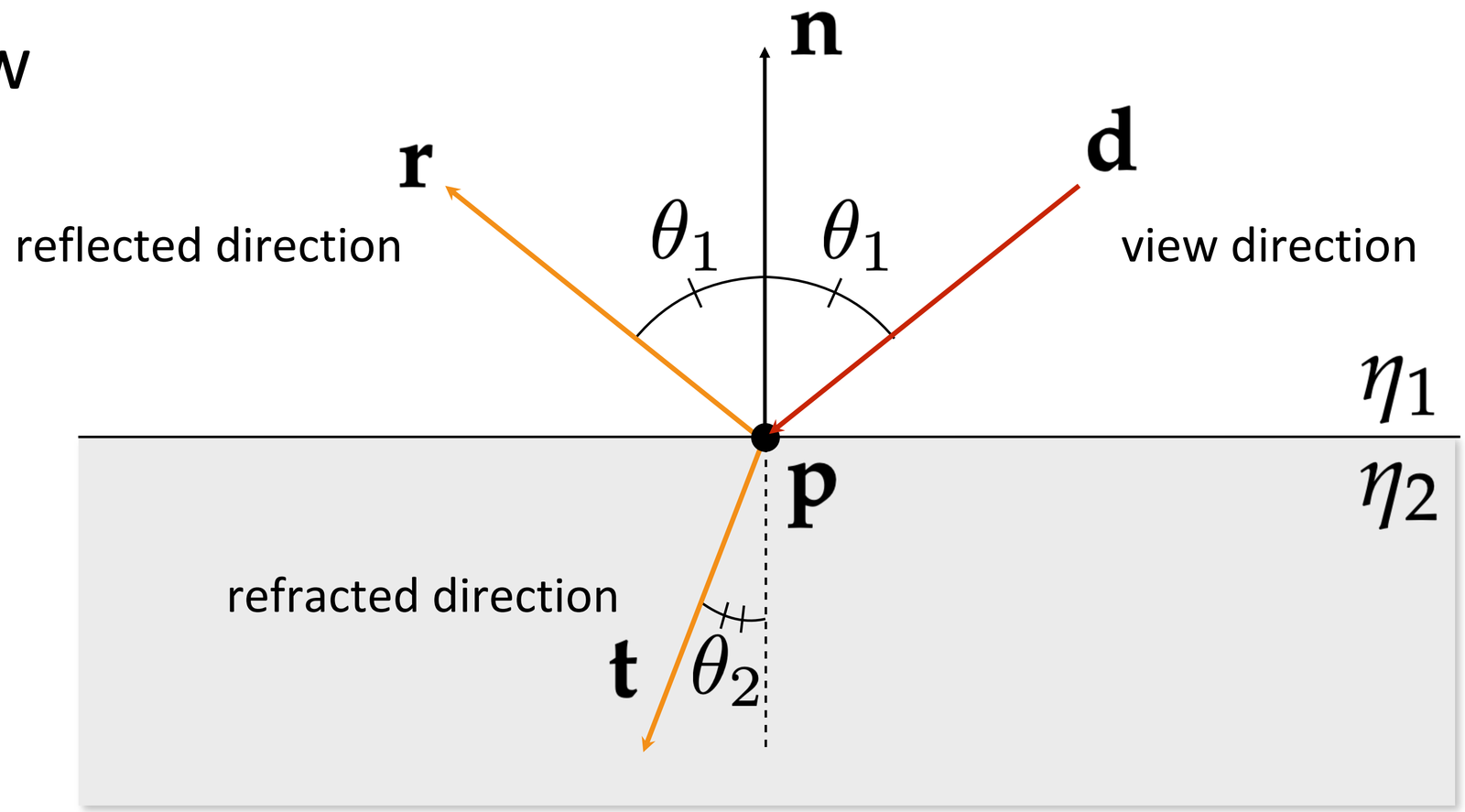
Snell's law



$$\eta_1 \sin\theta_1 = \eta_2 \sin\theta_2$$

Specular transmission/refraction

Snell's law



$$\mathbf{t} = \eta_1/\eta_2 \left(\mathbf{d} - (\mathbf{d}\cdot\mathbf{n})\,\mathbf{n}\right) - \mathbf{n}\sqrt{1 - \eta_1^2/\eta_2^2 \left(1 - (\mathbf{d}\cdot\mathbf{n})^2\right)}$$

What is this dark circle?



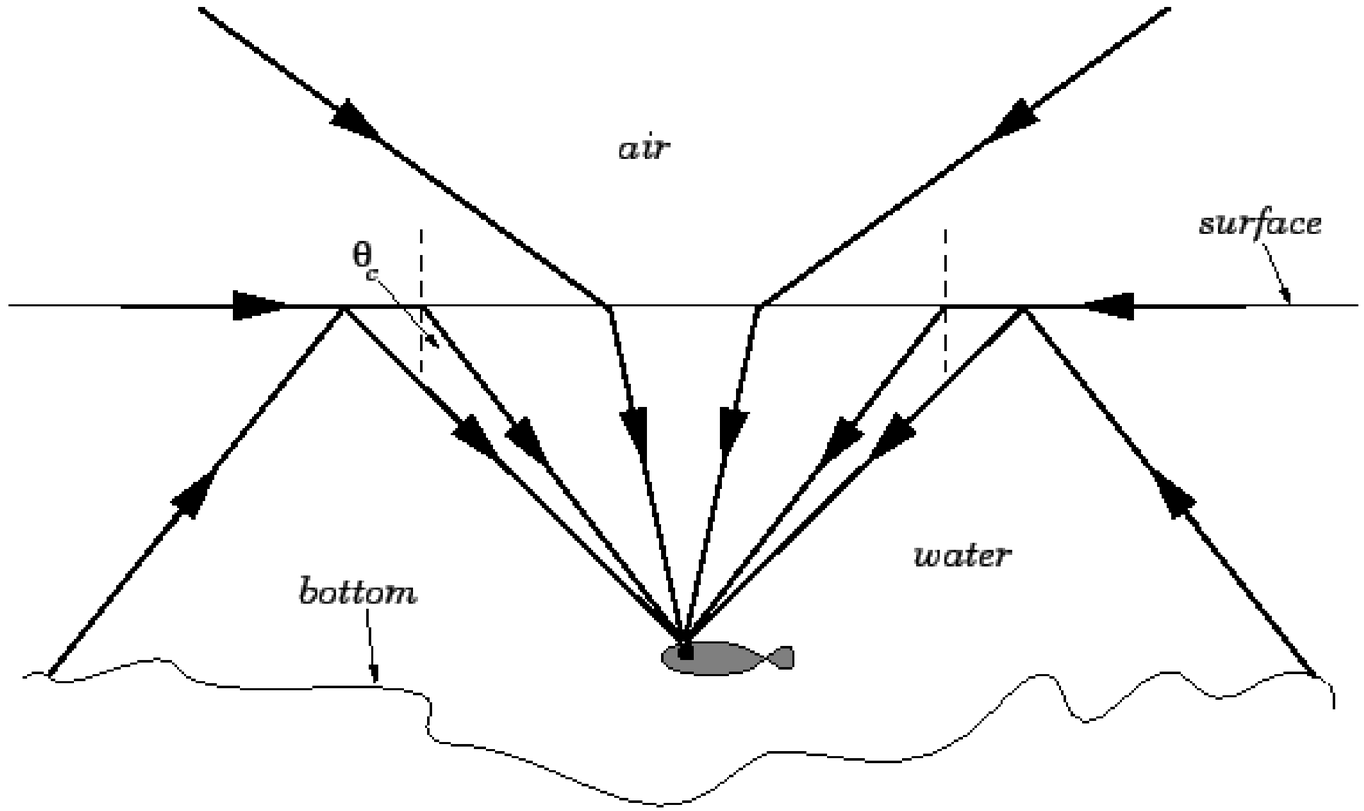
source: mrreid.org

What is this dark circle?



Called
“Snell's window”

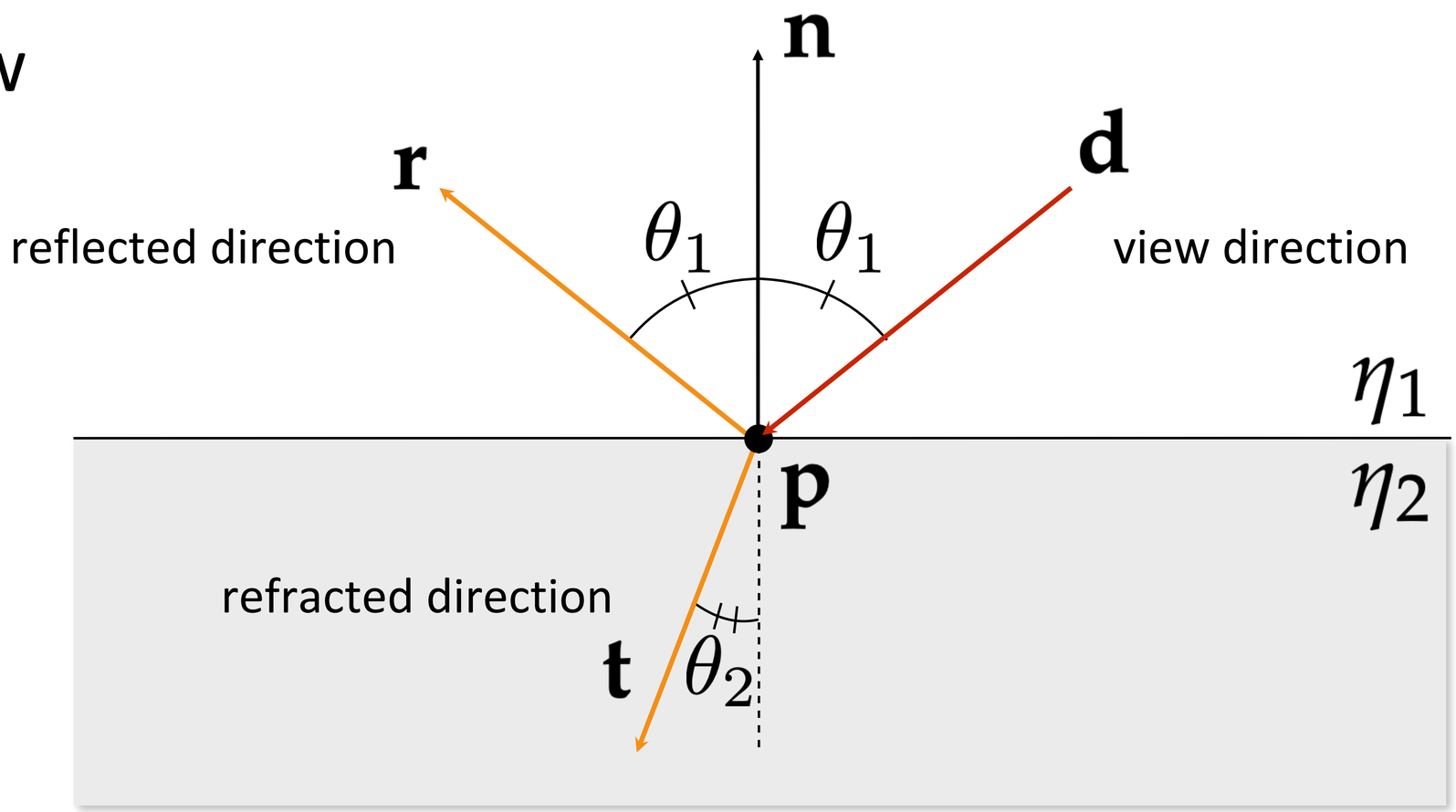
Caused by total
internal reflection



source: mrreid.org

Recall…

Snell's law



$$\mathbf{t} = \eta_1/\eta_2 \left(\mathbf{d} - (\mathbf{d}\cdot\mathbf{n})\,\mathbf{n}\right) - \mathbf{n}\sqrt{1 - \eta_1^2/\eta_2^2 \left(1 - (\mathbf{d}\cdot\mathbf{n})^2\right)}$$

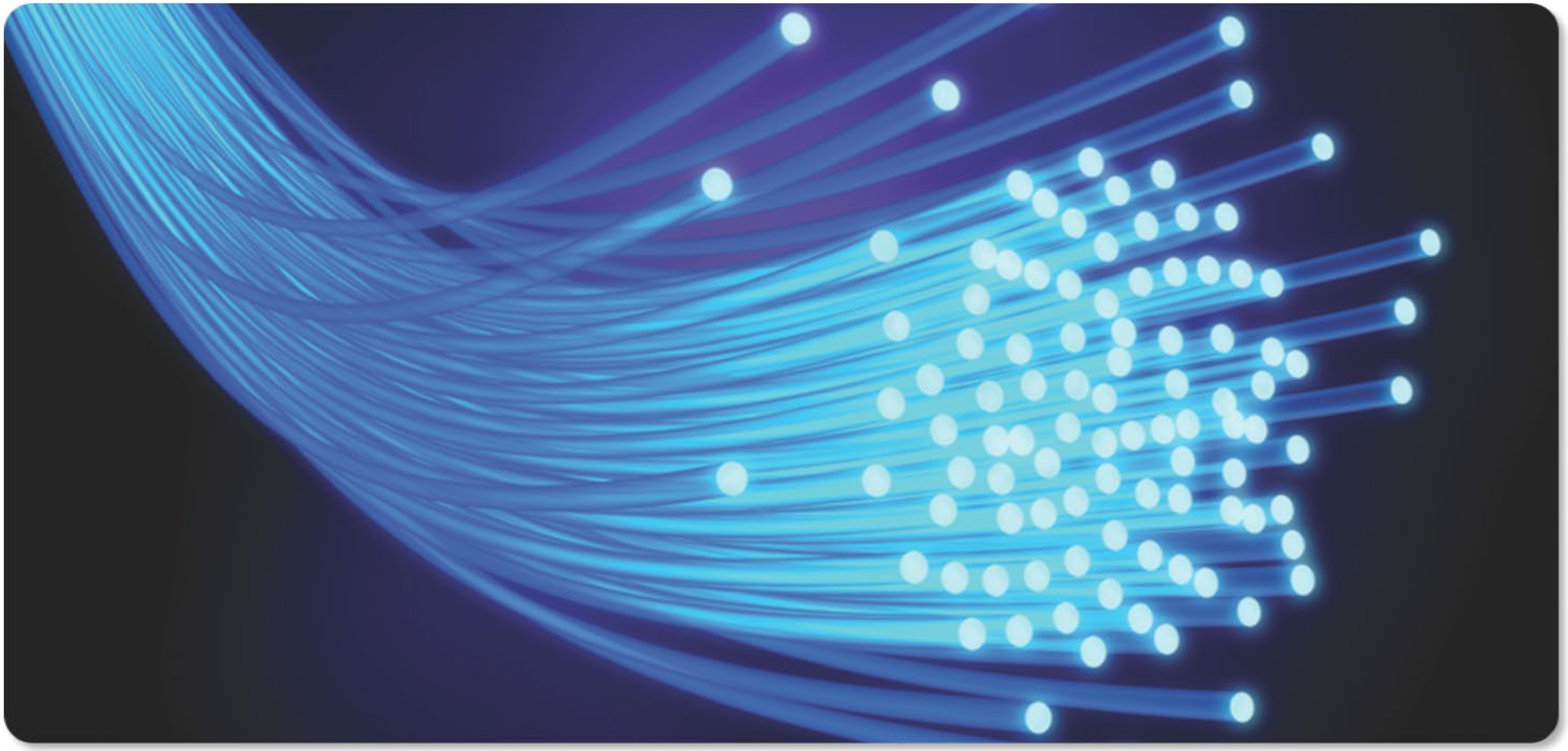
Can only happen when the ray starts in the higher index medium

Total Internal Reflection





Total Internal Reflection



Total Internal Reflection

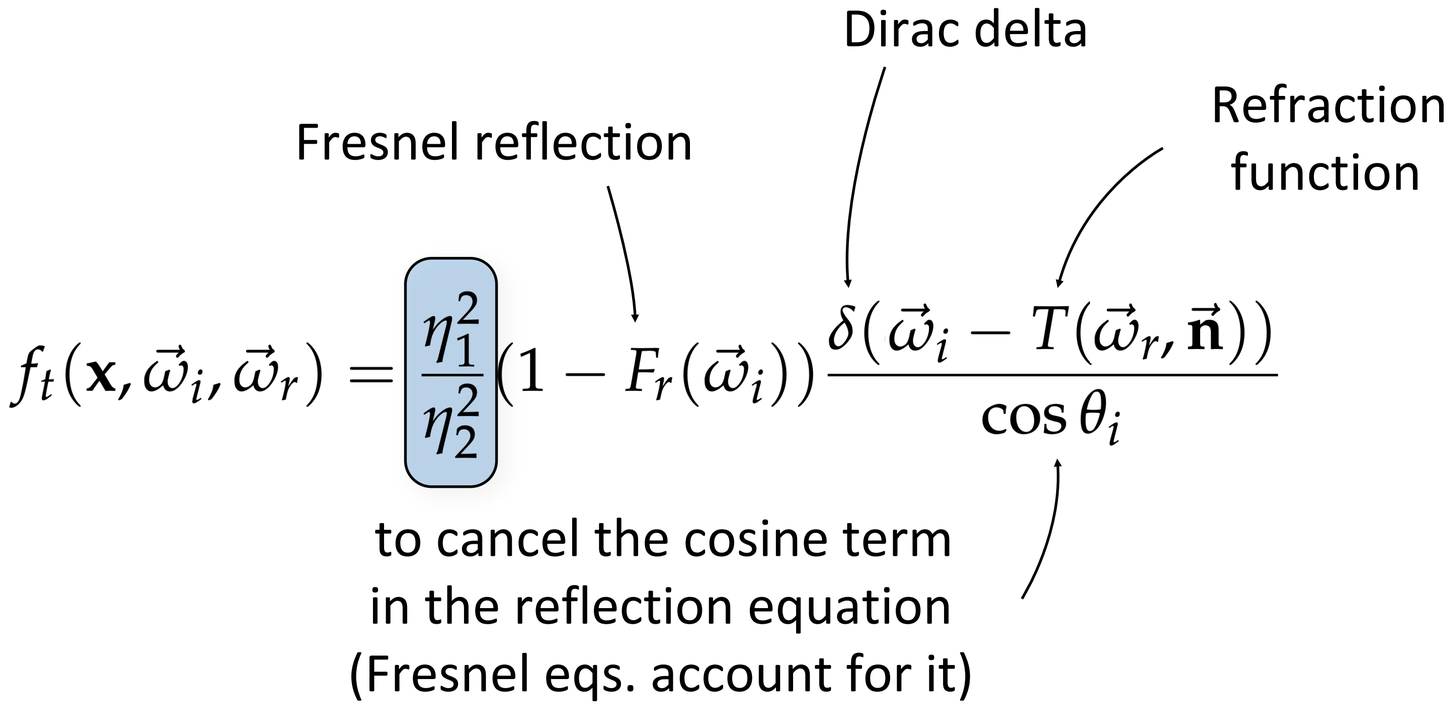


source: imgur.com

BTDF of Ideal Specular Refraction

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, \mathrm{d}\vec{\omega}_i$$

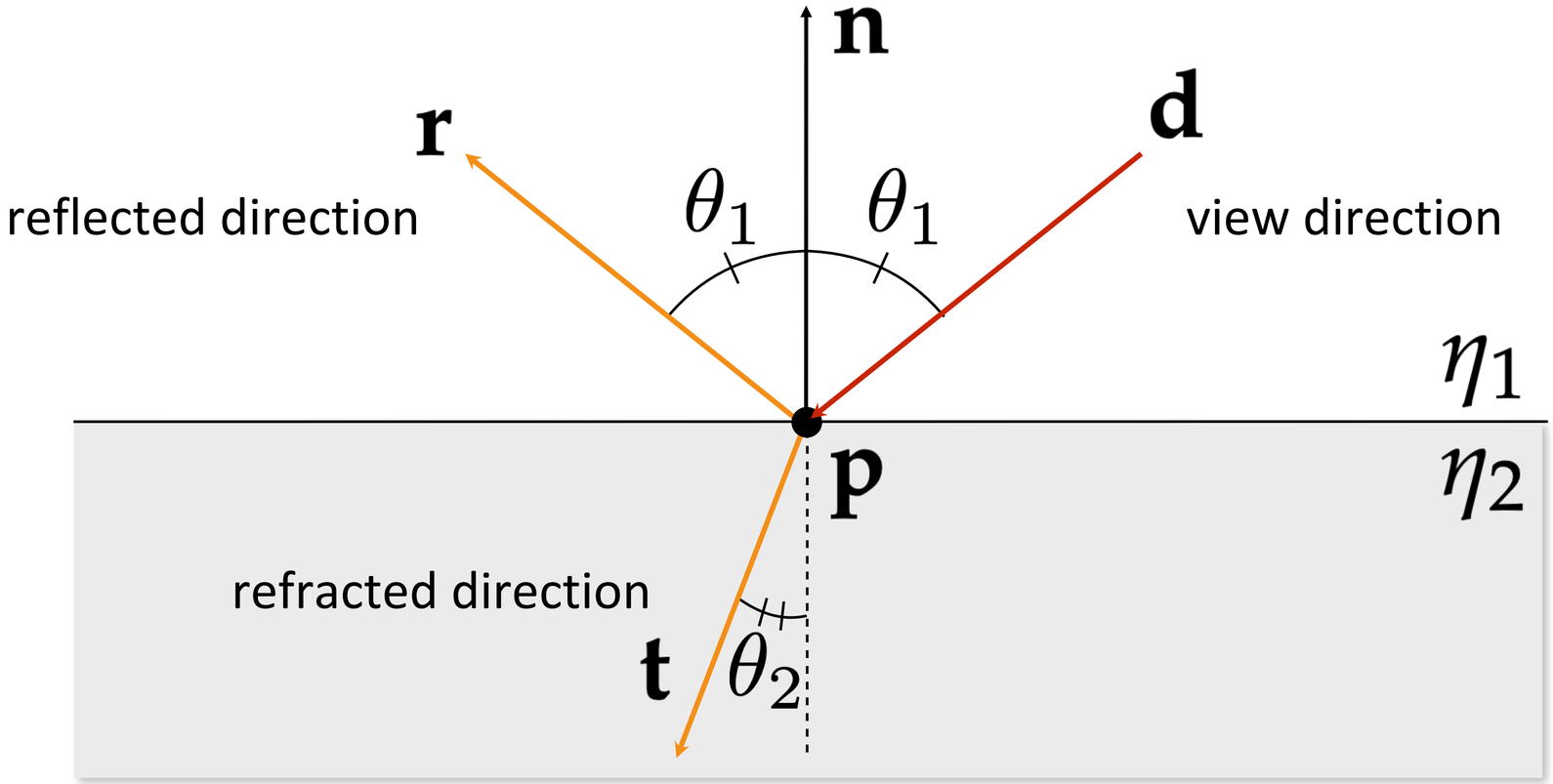
What is the BTDF for specular refraction?



Reflection vs. Refraction

How much light is reflected vs. refracted?

- in reality determined by “Fresnel equations”



Fresnel Equations

Reflection and *refraction* from smooth *dielectric* (e.g. glass) surfaces

Reflection from *conducting* (e.g. metal) surfaces

Derived from Maxwell equations

Involves polarization of the wave

Fresnel Equations for Dielectrics

Reflection of light polarized parallel and perpendicular to the plane of refraction

$$\rho_{||} = \frac{\eta_2 \cos\theta_1 - \eta_1 \cos\theta_2}{\eta_2 \cos\theta_1 + \eta_1 \cos\theta_2}$$

$$\rho_{\perp} = \frac{\eta_1 \cos\theta_1 - \eta_2 \cos\theta_2}{\eta_1 \cos\theta_1 + \eta_2 \cos\theta_2}$$

reflected: $F_r = \frac{1}{2}\left(\rho_{||}^2 + \rho_{\perp}^2\right)$

refracted: $F_t = 1 - F_r$

What’s happening in this photo?



source: flickr user neofob

Polarizing Filter



Polarization



Without Polarizer



With Polarizing Filter

source: photography.ca

Polarization



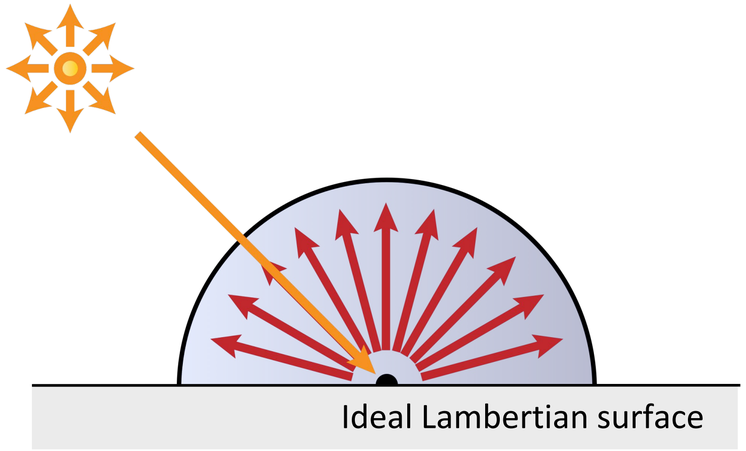
Without Polarizer

With Polarizing Filter

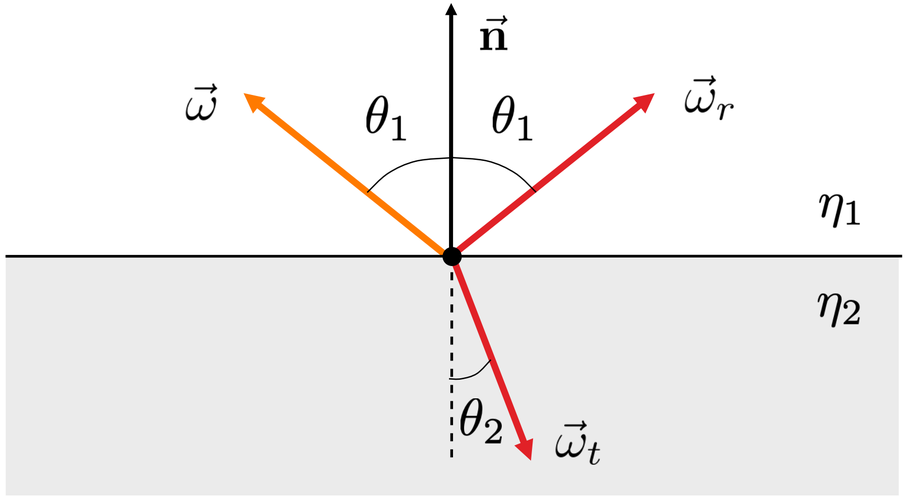
source: wikipedia

So Far: Idealized BRDF Models

Diffuse



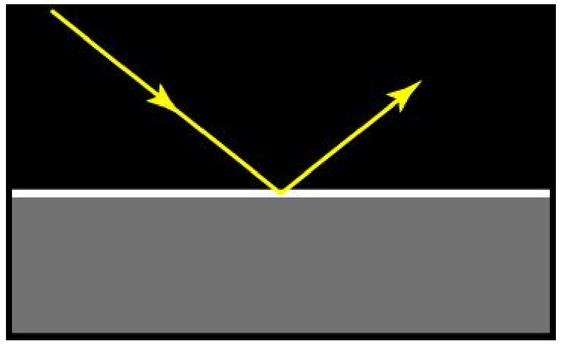
Specular Reflection and Refraction



Real-world materials

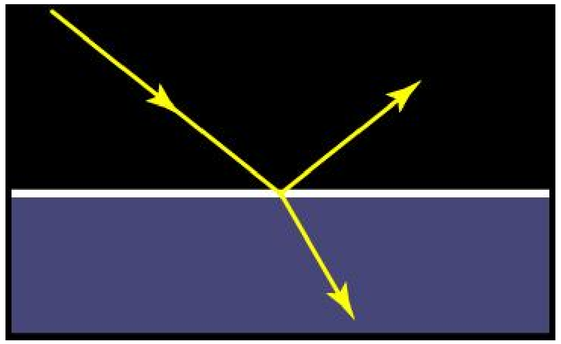
Metals





Dielectric

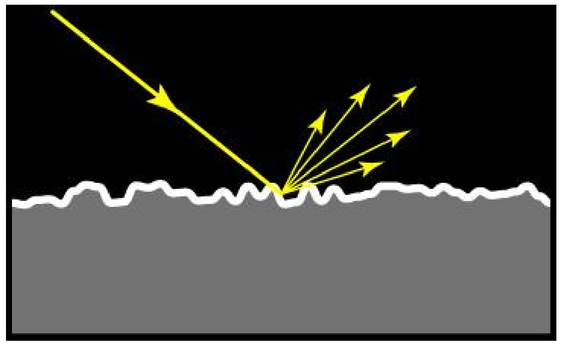




Real-world materials

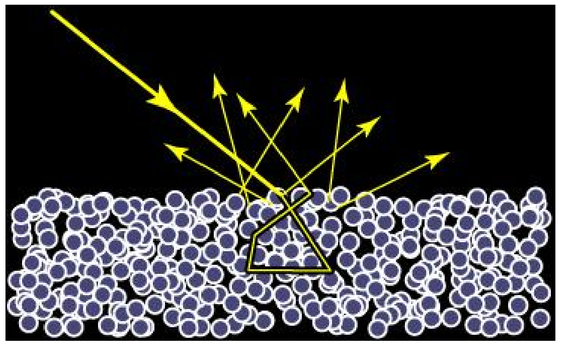
Metals

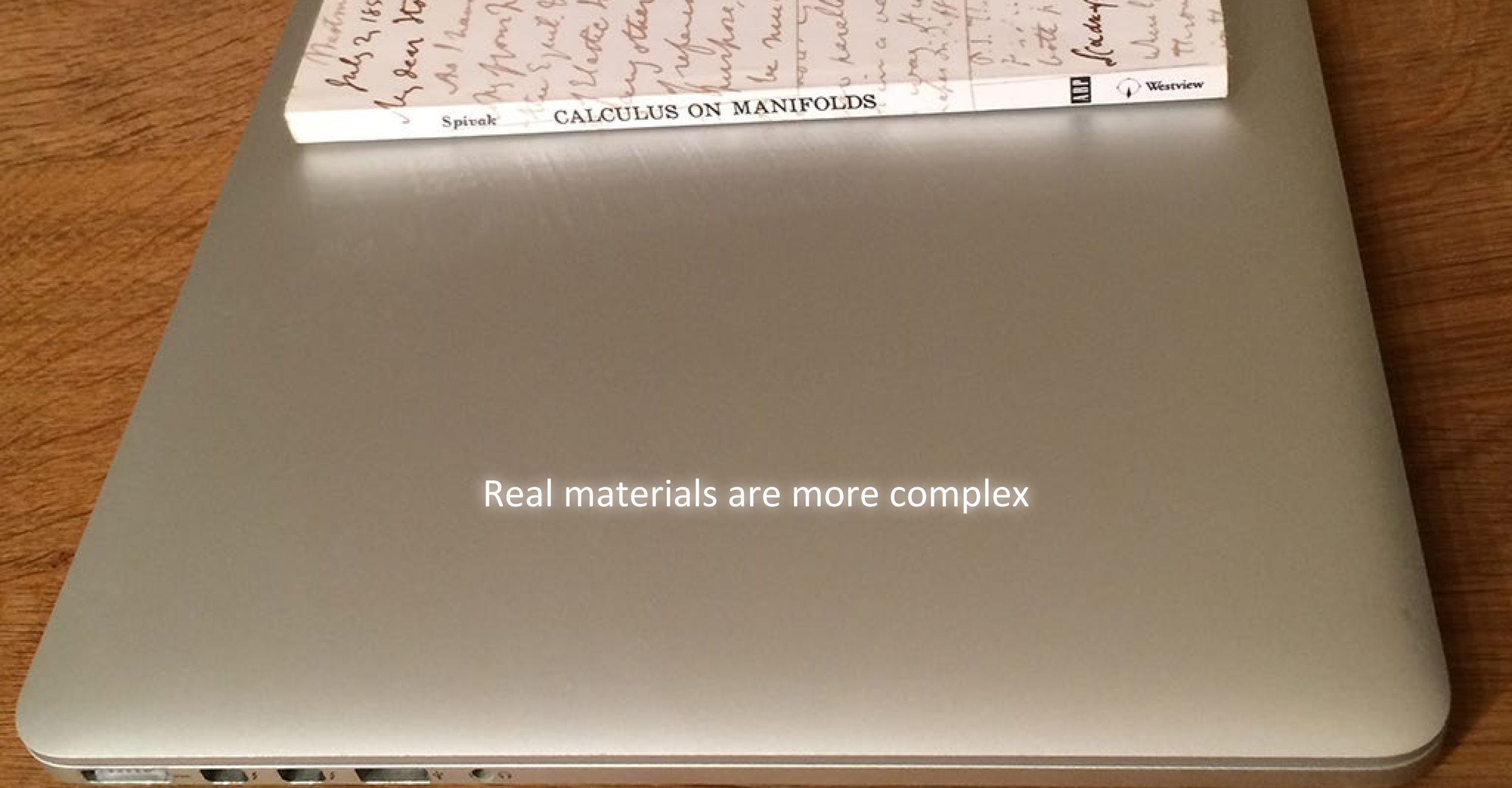




Dielectric







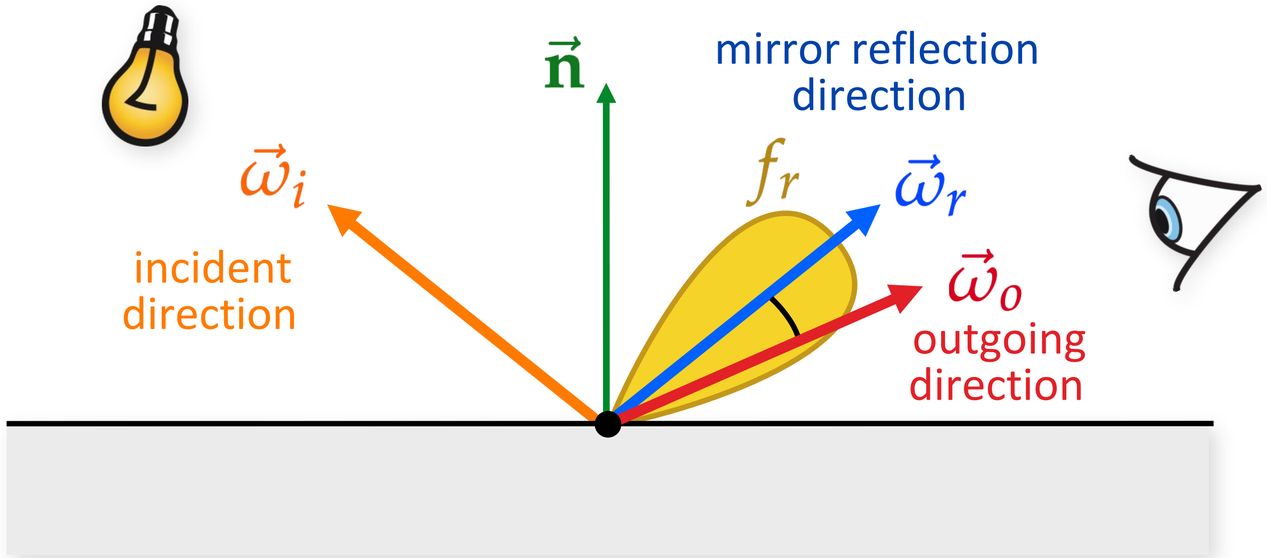
Real materials are more complex

Phong BRDF

Reflection direction distributed over an exponentiated cosine lobe:

$$f_r(\vec{\omega}_o, \vec{\omega}_i) = \frac{e+2}{2\pi}(\vec{\omega}_r \cdot \vec{\omega}_o)^e$$

$$\vec{\omega}_r = (2\vec{\mathbf{n}}(\vec{\mathbf{n}} \cdot \vec{\omega}_i) - \vec{\omega}_i)$$

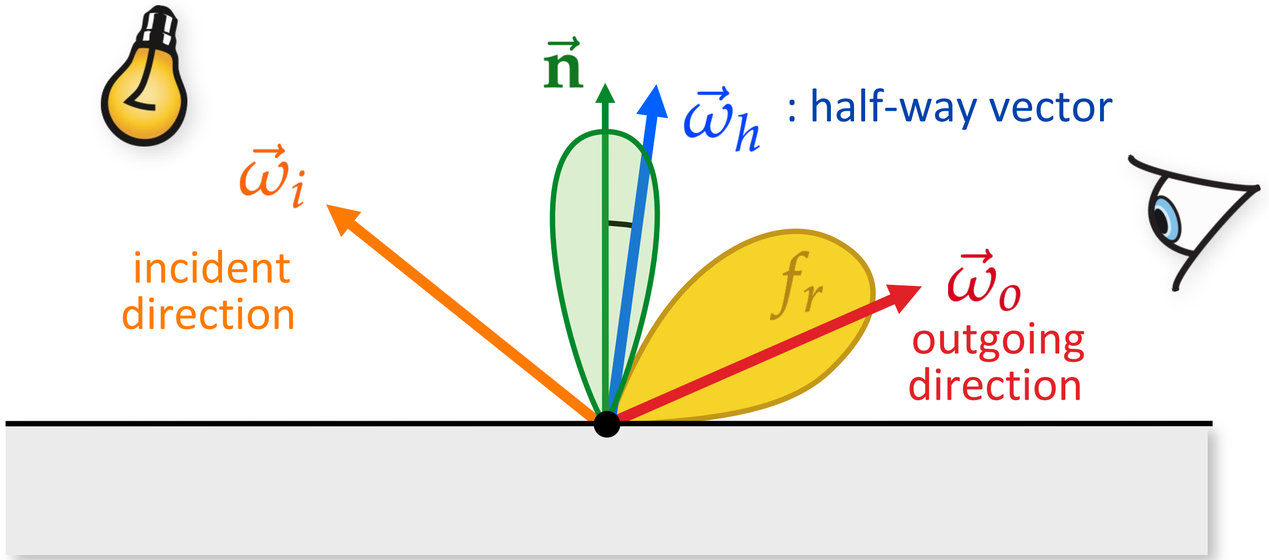


Blinn-Phong BRDF

Distribution of normals instead of reflection directions

$$f_r(\vec{\omega}_o, \vec{\omega}_i) = \frac{e+2}{2\pi}(\vec{\omega}_h \cdot \vec{\mathbf{n}})^e$$

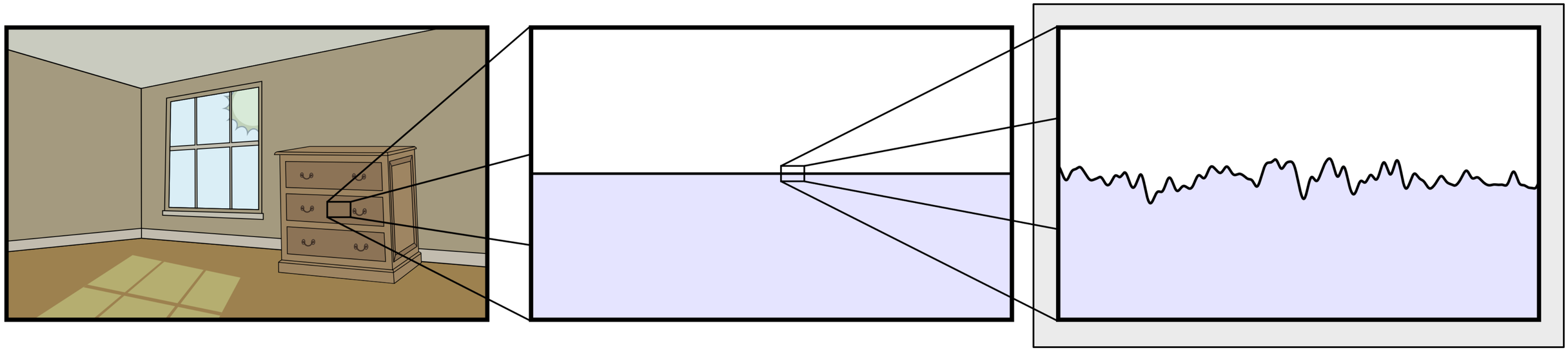
$$\vec{\omega}_h = \frac{\vec{\omega}_i + \vec{\omega}_o}{\|\vec{\omega}_i + \vec{\omega}_o\|}$$



Microfacet Theory

Key idea:

- transition from individual interactions to statistical averages



Macro scale

Scene geometry

Meso scale

Detail at intermediate scales

(can have variations here too)

Micro scale

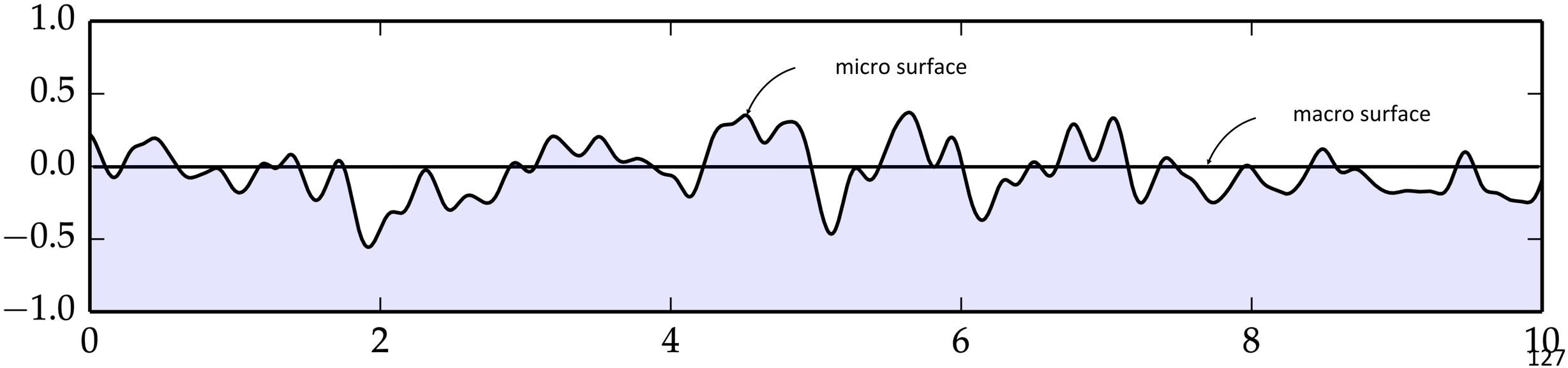
Roughness

Microfacet Theory

Assume surface consists of tiny facets

Assume that the differential area being viewed/illuminated is relatively large compared to the size of microfacets

A facet can be perfectly specular or diffuse



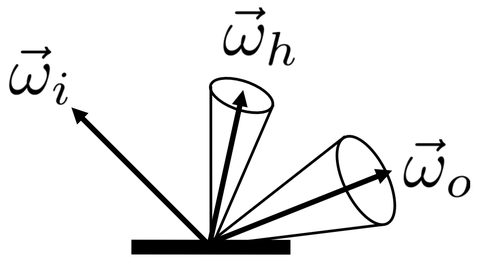
General Microfacet Model

Fresnel coefficient

Microfacet distribution

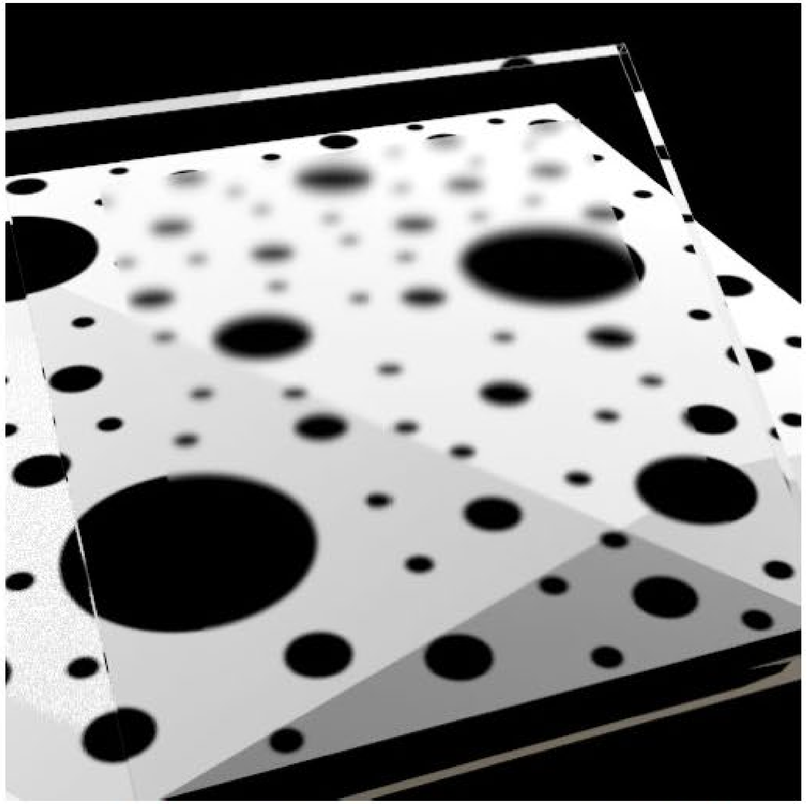
Shadowing/ masking

$$f(\vec{\omega}_i, \vec{\omega}_o) = \frac{F(\vec{\omega}_h, \vec{\omega}_o) \cdot D(\vec{\omega}_h) \cdot G(\vec{\omega}_i, \vec{\omega}_o)}{4|(\vec{\omega}_i \cdot \vec{\mathbf{n}})(\vec{\omega}_o \cdot \vec{\mathbf{n}})|}$$



$$\vec{\omega}_h = \frac{\vec{\omega}_i + \vec{\omega}_o}{\|\vec{\omega}_i + \vec{\omega}_o\|}$$

GGX and Beckmann



anti-glare (Beckman, $\alpha_b = 0.023$)



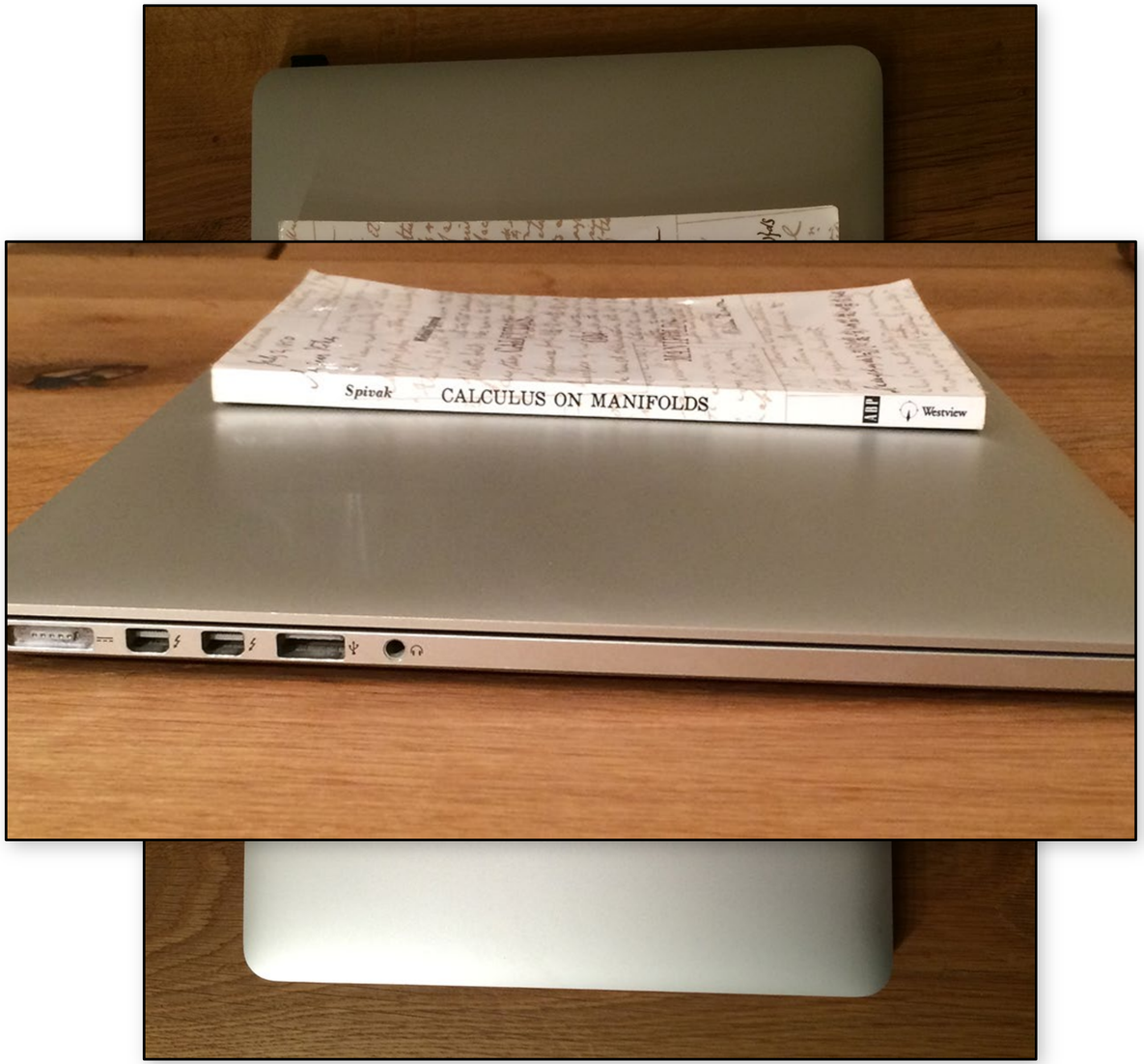
ground (GGX, $\alpha_g = 0.394$)

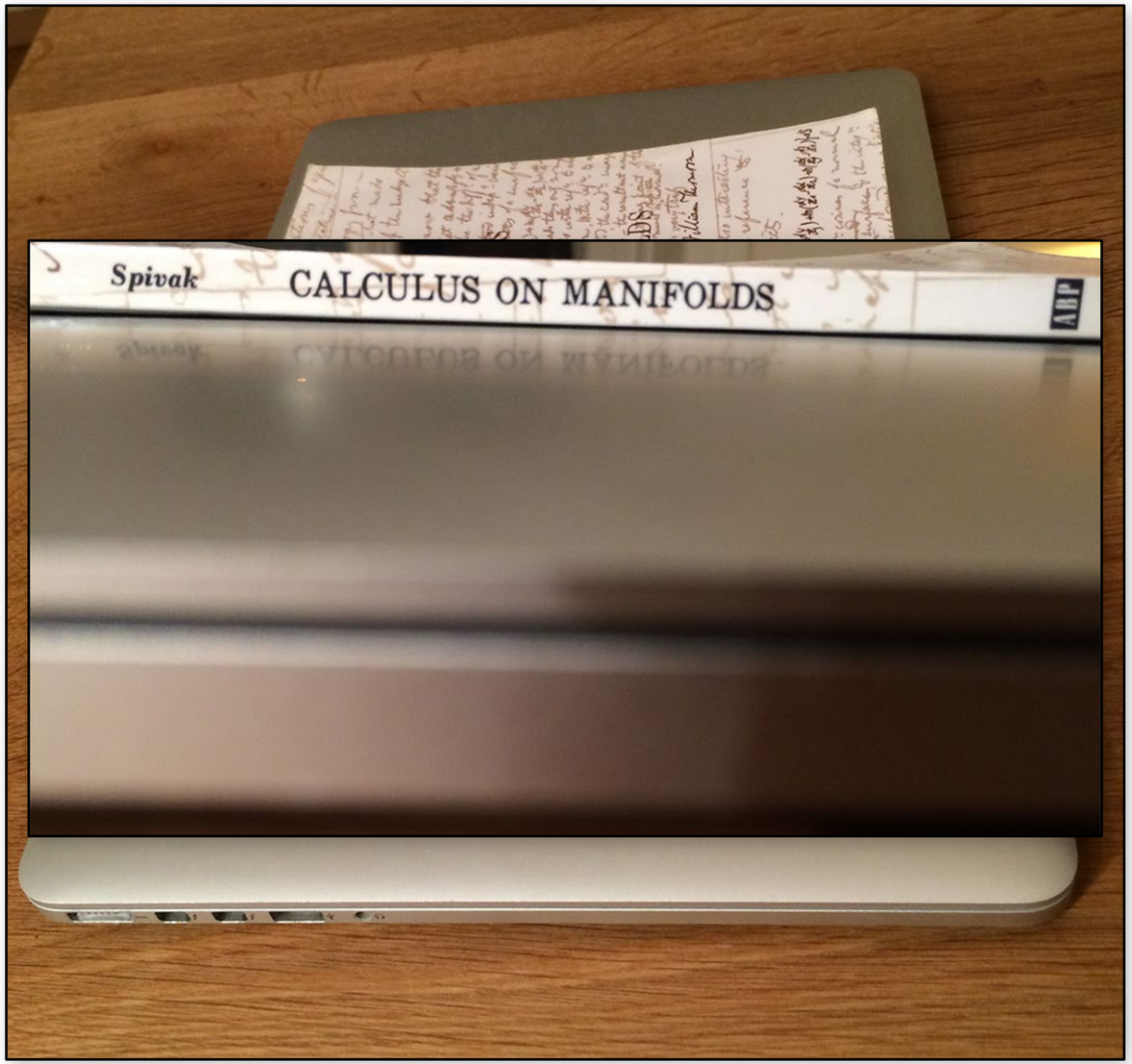


etched (GGX, $\alpha_g = 0.553$)

Walter et al. 07

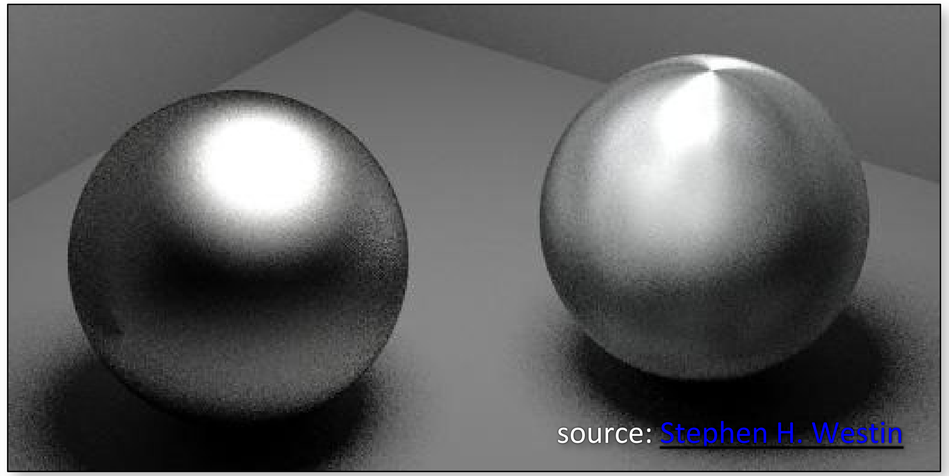
Interesting grazing angle behavior





Extension: Anisotropic Reflection







The Oren-Nayar Model

Same concept as the microfacet models, but assumes that the facets are diffuse

Shadowing/masking + interreflections

No analytic solution; fitted approximation

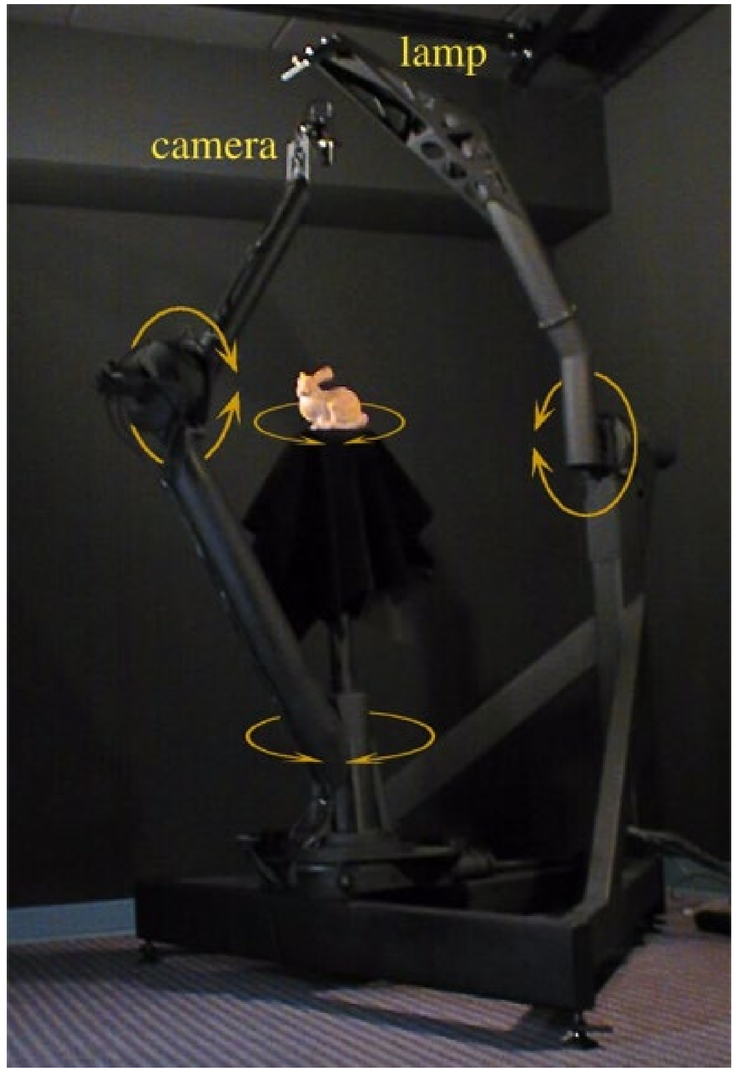
$$f_r(\vec{\omega}_o, \vec{\omega}_i) = \frac{\rho}{\pi}\left(A + B\ \max(0, \cos(\phi_i - \phi_o))\sin\alpha\tan\beta\right)$$

$$A = 1 - \frac{\sigma^2}{2(\sigma^2 + 0.33)} \qquad B = \frac{0.45\sigma^2}{\sigma^2 + 0.09}$$

$$\alpha = \max(\theta_i, \theta_o) \qquad \beta = \min(\theta_i, \theta_o)$$

Ideal Lambertian is just a special case (σ = 0)

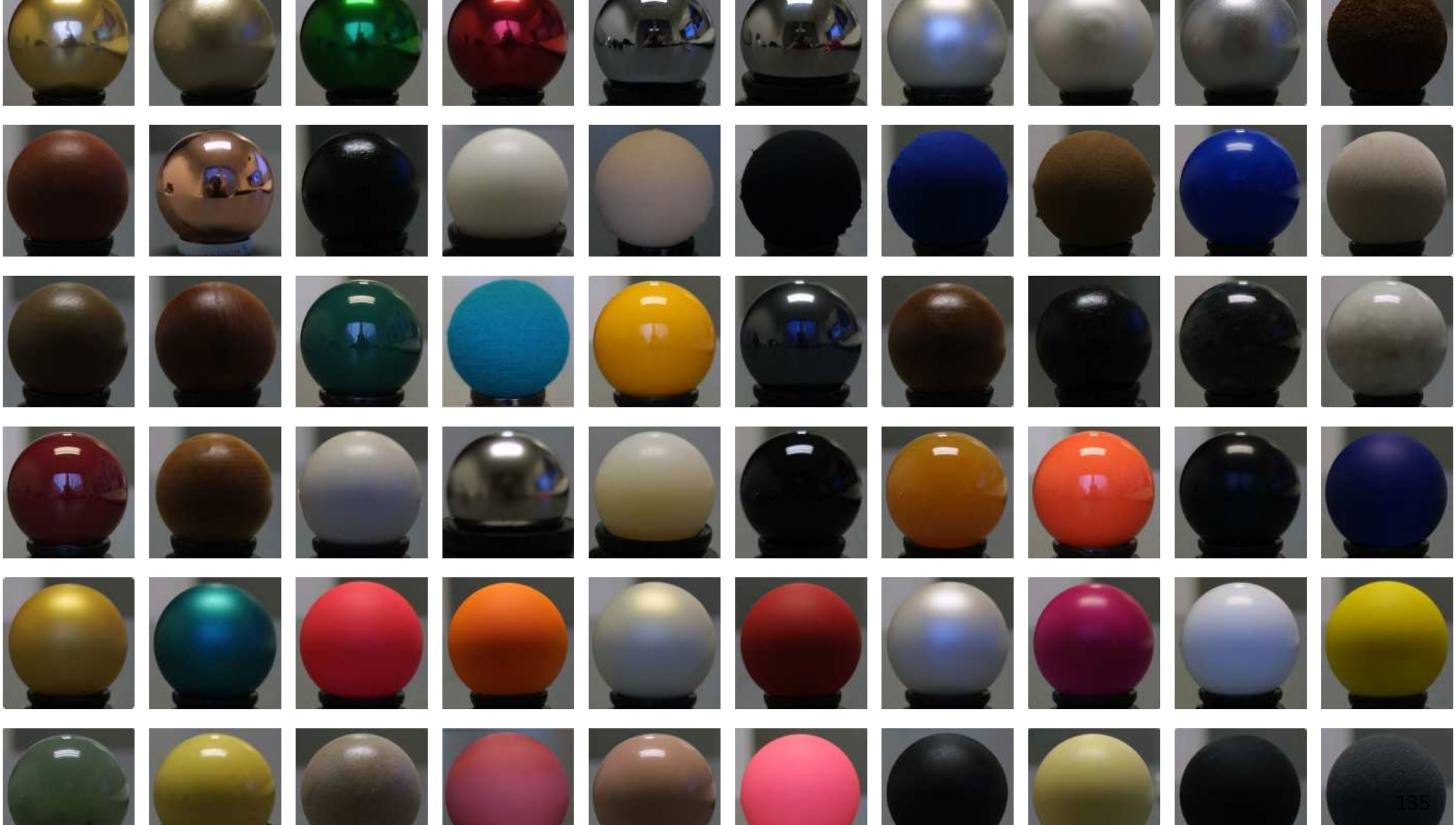
Measuring BRDFs

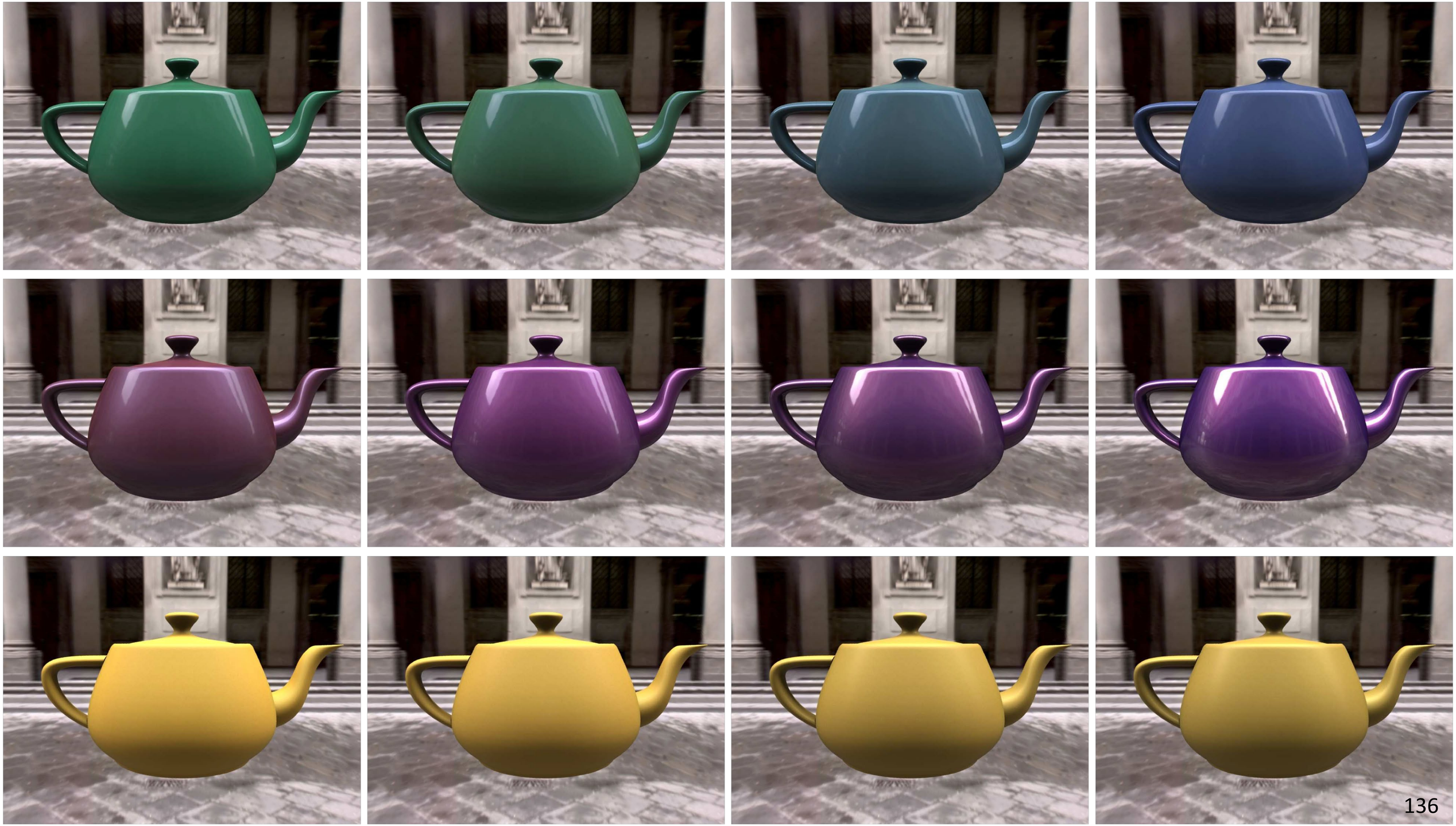


Measuring BRDFs



source: Matusik et al. 2003





The MERL Database

"A Data-Driven Reflectance Model"
Wojciech Matusik, Hanspeter Pfister, Matt Brand and Leonard McMillan.
ACM Transactions on Graphics 22, 3(2003), 759-769.

- http://www.merl.com/brdf/